Solution for AITS 3rd (Maths)-JEE-MAINS

-By S&K Classes

1. *If the roots of the equation $ax^2 + bx + c = 0$ are of the form $\frac{\alpha}{\alpha-1}$ and $\frac{\alpha+1}{\alpha}$, then value of $(a+b+c)^2$ is*

   (a) $2b^2 - ac$ (b) $b^2 - 2ac$ (c) $b^2 - 4ac$ (d) $4b^2 - 2ac$

Solution: *(c) By hypothesis* $\frac{\alpha}{\alpha-1} + \frac{\alpha+1}{\alpha} = -\frac{b}{a}$ and $\frac{\alpha}{\alpha-1}.\frac{\alpha+1}{\alpha} = \frac{c}{a}$

$\Rightarrow \frac{2\alpha^2-1}{\alpha^2-\alpha} = -\frac{b}{a}$ and $\alpha = \frac{c+a}{c-a}$

$\Rightarrow (c+a)^2 2b(c+a) + b^2 = b^2 - 4ac$

$\Rightarrow (a+b+c)^2 = b^2 - 4ac$

2. *The value of a, for which one root of the equation $(a-5)x^2 - 2ax + (a-4) = 0$ is smaller than 1 and the other is greater than 2 is*

   (a) $a \in (5, 24)$ (b) $a \in \left(\frac{20}{3}, \infty\right)$ (c) $a \in (5, \infty)$ (d) $(-\infty, \infty)$

Solution: *(a) (i) $D > 0$, $4a^2 - 4(a-5)(a-4) > 0$*

$9a - 20 > 0 \Rightarrow a > \frac{20}{9} \Rightarrow a \in \left(\frac{20}{9}, \infty\right)$ (i)

(ii) $(a-5) f(1) < 0$; $(a-5) f(2) < 0$

$\Rightarrow (a-5)(a-5-2a+a-4) < 0$

$\Rightarrow a > 5 \Rightarrow a \in (5, \infty)$ (ii)

$and\ (a-5)(a-24) < 0 \Rightarrow 5 < a < 24$

$\Rightarrow a \in (5, 24)$ *(iii)*

Using (1), (ii) & (iii)

The common condition is $a \in (5, 24)$

3. $\sin ax + \cos ax$ and $|\sin x| + |\cos x|$ are periodic of same fundamental period, if a equals

   [a] 0 [b] 1 [c] 2 [d] 4

Solution :- [d]

Period of $\sin ax$ is $\frac{2\pi}{a}$

And period of $\cos a\, x$ is $\frac{2\pi}{a}$

∴ Period of $\sin ax + \cos ax$ is $\frac{2\pi}{a}$

And period of $|\sin x| + |\cos x|$ is $\frac{\pi}{2}$

Given, $\frac{2\pi}{a} = \frac{\pi}{2}$

$\Rightarrow$ $a = 4$

4. If $f(2x+3y, 2x-7y) = 20x$, then $f(x,y)$ equals

   [a] $7x - 3y$ [b] $7x + 3y$ [c] $3x - 7y$ [d] $3x + 7y$

Solution :- [b]

Let $2x + 3y = A$ and $2x - 7y = B$

Then, $7A + 3B = 20x$

$\therefore \quad f(A,B) = 7A + 3B$

$\therefore \quad f(x,y) = 7x + 3y$

5. The solution of the equation $3^{\log_a x} + 3x^{\log_a 3} = 2$ is given by

[a] $a^{\log_3 a}$ [b] $\left(\frac{2}{a}\right)^{\log_3 2}$ [c] $a^{-\log_3 2}$ [d] $2^{-\log_3 a}$

Solution :-

$\Rightarrow 3^{\log_a x} + 3.x^{\log_a 3} = 2$

$\Rightarrow \quad 3^{\log_a x} + 3.3^{\log_a x} = 2$

$\Rightarrow \quad 4.3^{\log_a x} = 2$

$\Rightarrow \quad 3^{\log_a x} = \left(\frac{1}{2}\right)$

$\Rightarrow \log_a x = -\log_3 2 \Rightarrow x = a^{-\log_3 2} = a^{\log_3(2^{-1})}$

$= (2^{-1})^{\log_3 a} = 2^{-\log_3 a}$

6. Let $f(n) = 2\cos nx,\ \forall n \in N$, then $f(1)f(n+1) - f(n)$ is equal to

(a) $f(n+3)$ (b) $f(n+2)$ (c) $f(n+1)f(2)$ (d) $f(n+2)f(2)$

SOLUTION : (B) $f(n) = 2\cos nx$

$\Rightarrow f(1)f(n+1) - f(n)$

$= 4\cos xc\cos(n+1)x - 2\cos nx$

$= 2[2\cos(n+1)x\cos x - \cos nx]$

$= 2[\cos(n+2)x + \cos nx + \cos nx]$

$= 2\cos(n+2)x = f(n+2)$

7. In any ΔABC, if $\cot\frac{A}{2}, \cot\frac{B}{2}, \cot\frac{C}{2}$ are in A.P., then a, b, c are in

(a) A.P. (b) G.P. (c) H.P. (d) none of these

SOLUTION : (A) $\cot\frac{A}{2}, \cot\frac{B}{2}, \cot\frac{C}{2}$ are in A.P.

$\Rightarrow 2\cot\frac{B}{2} = \cot\frac{A}{2} + \cot\frac{C}{2}$

$\Rightarrow 2\sqrt{\frac{s(s-b)}{(s-a)(s-c)}}$

$= \sqrt{\frac{s(s-a)}{(s-b)(s-c)}} + \sqrt{\frac{s(s-c)}{(s-a)(s-b)}}$

$\Rightarrow 2(s-b) = s-a+s-c$

$\Rightarrow 2b = a+c$

$\Rightarrow a, b, c$ are in A.P.

8. If $(b^2-4ac)^2(1+4a^2) < 64a^2, a < 0$, then maximum value of quadratic expression ax^2+bx+c is always less than

(a) 0 (b) 2 (c) -1 (d) -2

SOLUTION : (B) $\frac{(b^2-4ac)^2}{16a^2} < \frac{4}{1+4a^2}$

Now,

$\max(ax^2+bx+c) = -\frac{b^2-4ac}{4a}$

also,

$\frac{-2}{\sqrt{1+4a^2}} < \frac{b^2-4ac}{4a} < \frac{2}{\sqrt{1+4a^2}}$ [from (1)]

So maximum value is always less than 2(when $a \to 0$).

9. The number of integral values of x satisfying $\sqrt{-x^2+10x-16} < x-2$ is
(a) 0 (b) 1 (c) 2 (d) 3

SOLUTION : (D) $\sqrt{-x^2+10x-16} < x-2$

We must have

$-x^2+10x-16 \geq 0$

$\Rightarrow x^2-10x+16 \leq 0$

$\Rightarrow 2 \leq x \leq 8$ (1)

Also,

$-x^2+10x-16 < x^2-4x+4$

$\Rightarrow 2x^2-14x+20 > 0$

$\Rightarrow \quad x^2-7x+10 > 0$

$\Rightarrow x^2-7x+10 > 0$

$\Rightarrow x > 5 \;\; or\, x < 2$

From (1) and (2)

$5 < x \leq 8 \Rightarrow x = 6, 7, 8$

10. If $z = x+iy$ and $x^2+y^2 = 16$, then the range of $||x|-|y||$ is
(a) [0, 4] (b) [0, 2] (c) [2, 4] (d) none of these

SOLUTION : (A) here $x = 4\cos\theta, y = 4\sin\theta$.

$\therefore \left||x|-|y|\right| = |4|\cos\theta|-4|\sin\theta||$

$=4||\cos\theta|-|\sin\theta||$

$= 4\sqrt{1-2|\cos\theta||\sin\theta|}$

$= 4\sqrt{1-|\sin 2\theta|}$

Hence, the range is [0, 4].

11. z and z_2 are two distinct points in an argand plane. If $a|z_1| = b|z_2|$ (where $a, b \in R$), then the point $(az_1/bz_2) + (bz_2/az_1)$ is a point on the
(a) Line segment [-2, 2] of the real axis (b) line segment [-2, 2] of the imaginary axis
(b) Unit circle |z|=1 (d) the line with $\arg z = tan^{-1}2$

SOLUTION : (A) assuming arg $z_1 = \theta$ and $\arg z_2 = \theta + a$,

$\frac{az_1}{bz_2} + \frac{bz_2}{az_1} = \frac{a|z_1|e^{i\theta}}{b|z_2|e^{i(\theta+\alpha)}} + \frac{b|z_2|e^{i(\theta+\alpha)}}{a|z_1|e^{i\theta}}$
$= e^{i\alpha} + e^{i\alpha} = 2\cos\alpha$

Hence , the point lies on the line segment [-2, 2] of the real axis.

12. If ax^3+bx^2+cx+d is divisible by ax^2+c, then a, b, c, d are in
(a) A.P. (b) G.P. (c) H.P. (d) none of these

SOLUTION : (D) since ax^3+bx^2+cx+d is divisible by ax^2+c, when ax^3+c the remainder should be zero.

if ax^3+bx^2+cx+d is divided by ax^2+c, then the remainder is (bc/a)- d . therefore,

$\frac{bc}{a} - d = 0$

$\Rightarrow bc = ad$

$\Rightarrow \frac{b}{a} = \frac{d}{c}$

Hence, from this , a , b, c, d are not necessarily in G.P.

13. The number of ordered pairs of integer (x, y) satisfying the equation $x^2+6x+y^2 = 4$ is
(a) 2 (b) 8 (c) 6 (d) none of these

SOLUTION : (B). $(x+3)^2+y^2=13$

$\Rightarrow x+3=\pm 2, y=\pm 3$ or $x+3=\pm 3, y=\pm 2$

14. If $f(3x+2)+f(3x+29)=0\ \forall\ x\in R$, then the period of $f(x)$ is

(a) 7 (b) 8 (c) 10 (d) none of these

SOLUTION : (D) $f(3x+2)+f(3x+29)=0$ (1)

Replacing x by $x+9$, we get

$f(3(x+9)+2)+f(3(x+9)+29)=0$

$\Rightarrow f(3x+29)+f(3x+56)=0$ (2)

From (1) and (2), we get

$f(3x+2)=f(3x+56)$

$\Rightarrow f(3x+2)=f(3(x+18)+2)$

$\Rightarrow f(x)$ is periodic with period 18.

15. If $(\cos\theta+i\sin\theta)(\cos 2\theta+i\sin 2\theta)\ ....\ (\cos n\theta+i\ \text{sni}\ n\theta)=1$, then the value of θ is:

(a) $4m\pi$ (b) $\frac{2m\pi}{n(n+1)}$ (c) $\frac{4m\pi}{n(n+1)}$ (d) $\frac{m\pi}{n(n+1)}$

Solution(c) we have $(\cos\theta+i\sin\theta)(\cos 2\theta+i\sin 2\theta)\ ....\ (\cos n\theta+i\ \text{sni}\ n\theta)=1$

$\Rightarrow \cos(\theta+2\theta+3\theta+\cdots+n\theta)+i\sin\theta\,(\theta+2\theta+\cdots+n\theta)=1$

$\Rightarrow \cos\left(\frac{n(n+1)}{2}\theta\right)+i\sin\left(\frac{n(n+1)}{2}\theta\right)=1$

$\cos\left(\frac{n(n+1)}{2}\theta\right)=1$ and $\sin\left(\frac{n(n+1)}{2}\theta\right)=0$

$\Rightarrow \frac{n(n+1)}{2}\theta=2m\pi$

$\Rightarrow \theta=\frac{4m\pi}{n(n+1)}$, where $m\in I$.

16. If $(1+i)(1+2i)(1+3i)\ldots(1+ni)=a+ib$, then $2.5.10\ldots(1+n^2)$ is equal to:

(a) a^2-b^2 (b) a^2+b^2 (c) $\sqrt{a^2+b^2}$ (d) $\sqrt{a^2-b^2}$

Solution: (b) we have $(1+i)(1+2i)(1+3i)\ldots(1+ni)=a+ib$ .... (i)

$\Rightarrow (1-i)(1-2i)(1-3i)\ldots(1-ni)=a-ib$ ... (ii)

Multiplying Eqs. (i) and (ii),

We get $2.5\ldots(1+n^2)=a^2+b^2$

17. Let z_1 and z_2 be nth roots of unity which are ends of a line segment that subtend a right angle at the origin. Then n must be of the form:

(a) 4k + 1 (b) 4k + 2 (c) 4k + 3 (d) 4k

Solution: (d) $1^{1/n}=\cos\frac{2r\pi}{n}+i\sin\frac{2r\pi}{n}$

Let $z_1=\cos\frac{2r_1\pi}{n}+i\sin\frac{2r_1\pi}{n}$

And $z_2=\cos\frac{2r_2\pi}{n}+i\sin\frac{2r_2\pi}{n}$

Then $\angle z_1 0 z_2=\text{amp}\left(\frac{z_1}{z_2}\right)=\text{amp}(z_1)-\text{amp}(z_2)=\frac{2(r_1-r_2)\pi}{n}=\frac{\pi}{2}$ (Given)

$\therefore\ n=4(r_1-r_2)=4\times$ integer, so n is of the form 4k.

18. Let z and ω be the two non-zero complex numbers such that $|z|=|\omega|$ and $\arg z+\arg\omega=\pi$. Then z is equal to:

(a) ω (b) $-\omega$ (c) $\bar{\omega}$ (d) $-\bar{\omega}$

Solution: (d) $\because |z|=|\omega|\Rightarrow |z|=|\bar{\omega}|$

$\arg z+\arg\omega=\pi\Rightarrow \arg z-\arg(\bar{\omega})=\pi$

$\therefore\ z+\bar{\omega}=0\Rightarrow z=-\bar{\omega}$

19. If z and ω are two non-zero complex numbers such that $|z\omega|=1$ and $\arg(z)-\arg(\omega)=\frac{\pi}{2}$, then $\bar{z}\omega$ is equal to:

(a) 1 (b) -1 (c) i (d) $-i$

Solution(d) $|z||\omega| = 1$ (i)

And $\arg\left(\frac{z}{\omega}\right) = \frac{\pi}{2} \Rightarrow \frac{z}{\omega} = i \Rightarrow \left|\frac{z}{\omega}\right| = 1$ (ii)

From equations (i) and (ii)

$|z| = |\omega| = 1$ and $\frac{z}{\omega} + \frac{\bar{z}}{\bar{\omega}} = 0; z\bar{\omega} + \bar{z}\omega = -i$

20. If $\left(\frac{1+i}{1-i}\right)^x = 1$, then:
(a) x = 4n, where n is any positive integer
(b) x = 2n, where n is any positive integer
(c) x = 4n + 1, where n is any positive integer
(d) x = 2n + 1, where n is any positive integer

Solution: (a) $\left(\frac{1+i}{1-i}\right)^x = 1 \Rightarrow \left[\frac{(1+i)^2}{1-i}\right]^x = 1$

$\Rightarrow \left(\frac{1+i^2+2i}{1+1}\right)^x = 1 \Rightarrow i^x = 1$

$\therefore x = 4n, n \in I^+.$

21. If z_1, z_2, z_3 are complex numbers such that $|z_1| = |z_2| = |z_3| = \left|\frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3}\right| = 1$, then $|z_1 + z_2 + z_3|$ is:
(a) Equals to 1 (b) Less than 1 (c) Greater than 3 (d) Equal to 3

Solution: (a) $1 = \left|\frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3}\right| = \left|\frac{z_1\overline{z_1}}{z_1} + \frac{z_2\overline{z_2}}{z_2} + \frac{z_3\overline{z_3}}{z_3}\right|$ $(\because |z_1|^2 = 1 = z_1\overline{z_1}, \text{etc})$

$= |\overline{z_1} + \overline{z_2} + \overline{z_3}| = |\overline{z_1 + z_2 + z_3}| = |z_1 + z_2 + z_3|$ $(\because |\overline{z_1}| = |z_1|)$

22. Let z, ω be complex numbers such that $z + \overline{i\omega} = 0$ and $\arg z\omega = \pi$, then $\arg z$ equals:

(a) $\frac{5\pi}{4}$ (b) $\frac{\pi}{2}$ (c) $\frac{3\pi}{4}$ (d) $\frac{\pi}{4}$

Solution: (c) Given that arg $z\omega = \pi$ (i)

$\bar{z} + i\,\overline{\omega} = 0 \Rightarrow \bar{z} = -i\,\overline{\omega}$

$\Rightarrow z = i\,\omega \Rightarrow \omega = -iz$

From Eq. (i), $\arg(-iz^2) = \pi$

Arg $(-i) + 2\arg(z) = \pi; \frac{-\pi}{2} + 2\arg(z) = \pi$

$2\arg(z) = \frac{3\pi}{2}; \arg(z) = \frac{3\pi}{4}$

23. The value of $\sum_{k=1}^{6}\left(\sin\frac{2\pi k}{7} - i\cos\frac{2\pi k}{7}\right)$ is:

(a) -1 (b) 0 (c) $-i$ (d) i

Solution: (d) $\sum_{k=1}^{6}\left(\sin\left(\frac{2\pi k}{7}\right) - i\cos\left(\frac{2\pi k}{7}\right)\right)$

$= -i\sum_{k=1}^{6}\left(\sin\frac{2\pi k}{7} + i\sin\frac{2\pi k}{7}\right)$

$= -i\sum_{k=1}^{6} e^{\frac{2\pi k}{7}}$

$= -i\left(\sum_{k=1}^{6} e^{\frac{2\pi k}{7}} - 1\right)$

$= -i(\text{Sum of 7, 7th roots of unity} - 1)$

$= -i(0 - 1) = i$

24. If z_1 and z_1 are two non-zero complex numbers such that $|z_1 + z_2| = |z_1| + |z_2|$, then $\arg(z_1) - \arg(z_2)$ is equal to:

(a) $-\pi$ (b) $-\frac{\pi}{2}$ (c) $\frac{\pi}{2}$ (d) 0

Solution: (d) Let $z_1 = r_1(\cos\theta_1 + i\sin\theta_1), z_2 = r_2(\cos\theta_2 + i\sin\theta_2)$

$\therefore |z_1 + z_2| = [(r_1\cos\theta_1 + r_2\cos\theta_2)^2 + (r_1\sin\theta_1 + r_2\sin\theta_2)^2]^{1/2}$

$= |r_1^2 + r_2^2 + 2r_1r_2\cos(\theta_1 - \theta_2)|^{1/2}$

$= [(r_1 + r_2)^2]^{1/2}$

$\therefore\ |z_1 + z_2| = |z_1| + |z_2|$

Therefore, $\cos(\theta_1 - \theta_2) = 1$

$\Rightarrow \quad \theta_1 - \theta_2 = 0 \Rightarrow \theta_1 = \theta_2$

Thus, $\arg(z_1) - \arg(z_2) = 0$

25. If $\omega = \alpha + i\beta$, where $\beta \neq 0$ and $z \neq 1$, satisfies the condition that $\left(\frac{\omega - \overline{\omega}z}{1-z}\right)$ is purely real, then the set of value of z is:

(a) $\{z: |z| = 1\}$ (b) $\{z: z = \overline{z}\}$ (c) $\{z: z \neq 1\}$ (d) $\{z: \overline{z} = 1. z \neq 1\}$

Solution: (d) Given $\left(\frac{\omega - \overline{\omega}z}{1-z}\right)$ is purely real $\Rightarrow z \neq 1$

$$\therefore \quad \left(\frac{\omega - \overline{\omega}z}{1-z}\right) = \overline{\left(\frac{\omega - \overline{\omega}z}{1-z}\right)}$$

$$= \frac{\overline{\omega} - \omega\overline{z}}{1-\overline{z}}$$

$\Rightarrow \quad (\omega - \overline{\omega}z)(1 - \overline{z}) = (1 - z)(\overline{\omega} - \omega\overline{z})$

$\Rightarrow \quad (z\overline{z} - 1)(\omega - \overline{\omega}) = 0$

$\Rightarrow \quad (|z|^2 - 1)(2i\beta) = 0 \qquad (\because \omega = \alpha + i\beta)$

$\therefore \quad |z|^2 - 1 = 0$

$\Rightarrow \quad |z| = 1$ and $z \neq 1 \qquad (\beta \neq o)$

26. A man walks a distance of 3 units from the origin towards the north-east (N 45^0 E) direction. From there, he walks a distance of 4 units towards the north-west (N 45^0 W) direction to reach a point P. then the position of P in the Argand Plane is:

(a) $3e^{i\pi/4} + 4i$ (b) $(3 - 4i)e^{i\pi/4}$ (c) $(4 + 3i)e^{i\pi/4}$ (d) $(3 + 4i)e^{i\pi/4}$

Solution: (d) Let OA = 3, so that the complex number associated with A is $3e^{i\pi/4}$. If z is the complex number associated with P, then

$$\frac{z - 3e^{i\pi/4}}{0 - 3e^{i\pi/4}} = \frac{4}{3}e^{i\pi/4} = -\frac{4i}{3}$$

$\Rightarrow \quad 3z - 9e^{i\pi/4} = 12ie^{i\pi/4}$

$\Rightarrow \quad z = (3 + 4i)e^{i\pi/4}$

27. If the quadratic equation

$$z^2 + (a + ib)z + c + id = 0$$

Where a, b, c, d are non-zero real numbers has a real root, then

(a) $abd = b^2c + d^2$ (b) $abd = bc^2 + d^2$

(c) $abd = bc^2 + ad^2$ (d) none of these

Solution : (a) Let the real number α be a root of $z^2 + (a + ib)z + c + id = 0$

$\Rightarrow \quad a^2 + (a + ib)a + c + id = 0$

$\Rightarrow \quad a^2 + a\alpha + c = 0$ and $b\alpha + d = 0$

Eliminating α we obtain

$$\left(-\frac{d}{b}\right)^2 + a\left(-\frac{d}{b}\right) + c = 0$$

$\Rightarrow \quad d^2 + abd + b^2c = 0 \quad \Rightarrow abd = bc^2 + d^2$

28. The locus of z, which satisfies the inequality :

$log_{0.3}|z = 1| > log_{0.3}|z - i|$ is given by :

(a) $x + y < 0$ (b) $x - y > 0$ (c) $x + y > 0$ (d) $x - y < 0$

Solution : (b) By the question ,$|z = 1| < |z - i|$

$\Rightarrow \quad |x + iy - 1| < |x + iy - i|$

$\Rightarrow \quad |(x - 1) + iy| < |x + i(y - 1)|$

$\Rightarrow \sqrt{(x-1)^2 + y^2} < \sqrt{x^2 + (y-1)^2}$

$\Rightarrow \quad (x-1)^2 + y^2 < x^2 + (y-1)^2$

$\Rightarrow x^2 - 2x + 1 + y^2 < x^2 + y^2 - 2y + 1$

$\Rightarrow \quad -2x < -2y \quad \Rightarrow -x < -y$

$\Rightarrow \quad 0 < x - y \quad \Rightarrow x - y > 0.$

29. If $\log_{\sqrt{3}} 5 = a \text{ and } \log_{\sqrt{3}} 2 = b$, then $\log_{\sqrt{3}} 300 =$

[a] $2(a+b)$ [b] $2(a+b+1)$ [c] $2(a+b+2)$ [d] $a+b+4$

Solution :- [c]

$\log_{\sqrt{3}} 300 = \log_{\sqrt{3}}(3 \times 2^2 \times 5^2)$

$= \log_{\sqrt{3}} 3 + 2\log_{\sqrt{3}} 2 + 2\log_{\sqrt{3}} 5$

$= 2\log_3 3 + 2b + 2a$

$(\because \log_{\sqrt{3}} 5 = a \text{ and } \log_{\sqrt{3}} 2 = b)$

$= 2(a+b+1)$

30. The function $f(x) = \sin\left(\log\left(x + \sqrt{(x^2+1)}\right)\right)$ is :

[a] Even function [b] odd function

[c] Neither even nor odd [d] Periodic function

Solution :- [b]

$f(x) = \sin\left(\log\left(x + \sqrt{1 + x^2}\right)\right)$

$\Rightarrow \quad f(-x) = \sin\left[\log\left(-x + \sqrt{1+x^2}\right)\right]$

$\Rightarrow \quad f(-x) = \sin\log\left(\left(\sqrt{1+x^2} - x\right)\frac{(\sqrt{1+x^2}+x)}{(\sqrt{1+x^2}+x)}\right)$

$\Rightarrow \quad f(-x) = \sin\log\left[\frac{1}{(x+\sqrt{1+x^2})}\right]$

$\Rightarrow \quad f(-x) = \sin\left[\log\left(x + \sqrt{1+x^2}\right)^{-1}\right]$

$\Rightarrow \quad f(-x) = \sin\left[-\log\left(x + \sqrt{1+x^2}\right)\right]$

$\Rightarrow \quad f(-x) = -\sin\left[\log\left(x + \sqrt{1+x^2}\right)\right]$

$\Rightarrow \quad f(-x) = -f(x)$

$\therefore f(x)$ is odd function.