

# Solution for AITS 3<sup>rd</sup> (Maths)-JEE-MAINS

## -By S&K Classes

1. If the roots of the equation  $ax^2 + bx + c = 0$  are of the form  $\frac{\alpha}{\alpha-1}$  and  $\frac{\alpha+1}{\alpha}$ , then value of  $(a + b + c)^2$  is  
 (a)  $2b^2 - ac$                       (b)  $b^2 - 2ac$                       (c)  $b^2 - 4ac$                       (d)  $4b^2 - 2ac$

**Solution:** (c) By hypothesis  $\frac{\alpha}{\alpha-1} + \frac{\alpha+1}{\alpha} = -\frac{b}{a}$  and  $\frac{\alpha}{\alpha-1} \cdot \frac{\alpha+1}{\alpha} = \frac{c}{a}$

$$\Rightarrow \frac{2\alpha^2 - 1}{\alpha^2 - \alpha} = -\frac{b}{a} \text{ and } \alpha = \frac{c+a}{c-a}$$

$$\Rightarrow (c+a)^2 2b(c+a) + b^2 = b^2 - 4ac$$

$$\Rightarrow (a+b+c)^2 = b^2 - 4ac$$

2. The value of  $a$ , for which one root of the equation  $(a-5)x^2 - 2ax + (a-4) = 0$  is smaller than 1 and the other is greater than 2 is

- (a)  $a \in (5, 24)$                       (b)  $a \in \left(\frac{20}{3}, \infty\right)$                       (c)  $a \in (5, \infty)$                       (d)  $(-\infty, \infty)$

**Solution:** (a) (i)  $D > 0$ ,  $4a^2 - 4(a-5)(a-4) > 0$

$$9a - 20 > 0 \Rightarrow a > \frac{20}{9} \Rightarrow a \in \left(\frac{20}{9}, \infty\right) \quad (i)$$

$$(ii) (a-5)f(1) < 0; (a-5)f(2) < 0$$

$$\Rightarrow (a-5)(a-5-2a+a-4) < 0$$

$$\Rightarrow a > 5 \Rightarrow a \in (5, \infty) \quad (ii)$$

$$\text{and } (a-5)(a-24) < 0 \Rightarrow 5 < a < 24$$

$$\Rightarrow a \in (5, 24) \quad (iii)$$

Using (i), (ii) & (iii)

The common condition is  $a \in (5, 24)$

3.  $\sin ax + \cos ax$  and  $|\sin x| + |\cos x|$  are periodic of same fundamental period, if  $a$  equals  
 [a] 0                      [b] 1                      [c] 2                      [d] 4

**Solution :- [d]**

Period of  $\sin ax$  is  $\frac{2\pi}{a}$

And period of  $\cos ax$  is  $\frac{2\pi}{a}$

$\therefore$  Period of  $\sin ax + \cos ax$  is  $\frac{2\pi}{a}$

And period of  $|\sin x| + |\cos x|$  is  $\frac{\pi}{2}$

Given,  $\frac{2\pi}{a} = \frac{\pi}{2}$

$\Rightarrow a = 4$

4. If  $f(2x + 3y, 2x - 7y) = 20x$ , then  $f(x, y)$  equals

- [a]  $7x - 3y$                       [b]  $7x + 3y$                       [c]  $3x - 7y$                       [d]  $3x + 7y$

**Solution :- [b]**

Let  $2x + 3y = A$  and  $2x - 7y = B$

Then,  $7A + 3B = 20x$

$$\therefore f(A, B) = 7A + 3B$$

$$\therefore f(x, y) = 7x + 3y$$

5. The solution of the equation  $3^{\log_a x} + 3x^{\log_a 3} = 2$  is given by

[a]  $a^{\log_3 a}$                       [b]  $\left(\frac{2}{a}\right)^{\log_3 2}$                       [c]  $a^{-\log_3 2}$                       [d]  $2^{-\log_3 a}$

**Solution :-**

$$\Rightarrow 3^{\log_a x} + 3 \cdot x^{\log_a 3} = 2$$

$$\Rightarrow 3^{\log_a x} + 3 \cdot 3^{\log_a x} = 2$$

$$\Rightarrow 4 \cdot 3^{\log_a x} = 2$$

$$\Rightarrow 3^{\log_a x} = \left(\frac{1}{2}\right)$$

$$\Rightarrow \log_a x = -\log_3 2 \Rightarrow x = a^{-\log_3 2} = a^{\log_3 (2^{-1})}$$

$$= (2^{-1})^{\log_3 a} = 2^{-\log_3 a}$$

6. Let  $f(n) = 2 \cos nx$ ,  $\forall n \in N$ , then  $f(1)f(n+1) - f(n)$  is equal to

(a)  $f(n+3)$                       (b)  $f(n+2)$                       (c)  $f(n+1)f(2)$                       (d)  $f(n+2)f(2)$

**SOLUTION : (B)**  $f(n) = 2 \cos nx$

$$\Rightarrow f(1)f(n+1) - f(n)$$

$$= 4 \cos x \cos(n+1)x - 2 \cos nx$$

$$= 2[2 \cos(n+1)x \cos x - \cos nx]$$

$$= 2[\cos(n+2)x + \cos nx + \cos nx]$$

$$= 2 \cos(n+2)x = f(n+2)$$

7. In any  $\triangle ABC$ , if  $\cot \frac{A}{2}$ ,  $\cot \frac{B}{2}$ ,  $\cot \frac{C}{2}$  are in A.P., then a, b, c are in

(a) A.P.                      (b) G.P.                      (c) H.P.                      (d) none of these

**SOLUTION : (A)**  $\cot \frac{A}{2}$ ,  $\cot \frac{B}{2}$ ,  $\cot \frac{C}{2}$  are in A.P.

$$\Rightarrow 2 \cot \frac{B}{2} = \cot \frac{A}{2} + \cot \frac{C}{2}$$

$$\Rightarrow 2 \sqrt{\frac{s(s-b)}{(s-a)(s-c)}}$$

$$= \sqrt{\frac{s(s-a)}{(s-b)(s-c)}} + \sqrt{\frac{s(s-c)}{(s-a)(s-b)}}$$

$$\Rightarrow 2(s-b) = s - a + s - c$$

$$\Rightarrow 2b = a + c$$

$\Rightarrow a, b, c$  are in A.P.

8. If  $(b^2 - 4ac)^2(1 + 4a^2) < 64a^2, a < 0$ , then maximum value of quadratic expression  $ax^2 + bx + c$  is always less than

(a) 0                      (b) 2                      (c) -1                      (d) -2

**SOLUTION : (B)**  $\frac{(b^2 - 4ac)^2}{16a^2} < \frac{4}{1 + 4a^2}$

Now,

$$\max(ax^2 + bx + c) = -\frac{b^2 - 4ac}{4a}$$

also,

$$\frac{-2}{\sqrt{1+4a^2}} < \frac{b^2-4ac}{4a} < \frac{2}{\sqrt{1+4a^2}} \quad [\text{from (1)}]$$

So maximum value is always less than 2 (when  $a \rightarrow 0$ ).

9. The number of integral values of  $x$  satisfying  $\sqrt{-x^2 + 10x - 16} < x - 2$  is  
 (a) 0 (b) 1 (c) 2 (d) 3

**SOLUTION : (D)**  $\sqrt{-x^2 + 10x - 16} < x - 2$

We must have

$$-x^2 + 10x - 16 \geq 0$$

$$\Rightarrow x^2 - 10x + 16 \leq 0$$

$$\Rightarrow 2 \leq x \leq 8 \quad (1)$$

Also,

$$-x^2 + 10x - 16 < x^2 - 4x + 4$$

$$\Rightarrow 2x^2 - 14x + 20 > 0$$

$$\Rightarrow x^2 - 7x + 10 > 0$$

$$\Rightarrow x^2 - 7x + 10 > 0$$

$$\Rightarrow x > 5 \text{ or } x < 2$$

From (1) and (2)

$$5 < x \leq 8 \Rightarrow x = 6, 7, 8$$

10. If  $z = x + iy$  and  $x^2 + y^2 = 16$ , then the range of  $||x| - |y||$  is  
 (a)  $[0, 4]$  (b)  $[0, 2]$  (c)  $[2, 4]$  (d) none of these

**SOLUTION : (A)** here  $x = 4 \cos \theta, y = 4 \sin \theta$ .

$$\therefore ||x| - |y|| = |4|\cos \theta| - 4|\sin \theta||$$

$$= 4||\cos \theta| - |\sin \theta||$$

$$= 4\sqrt{1 - 2|\cos \theta||\sin \theta|}$$

$$= 4\sqrt{1 - |\sin 2\theta|}$$

Hence, the range is  $[0, 4]$ .

11.  $z_1$  and  $z_2$  are two distinct points in an argand plane. If  $a|z_1| = b|z_2|$  (where  $a, b \in \mathbb{R}$ ), then the point  $(az_1/bz_2) + (bz_2/az_1)$  is a point on the  
 (a) Line segment  $[-2, 2]$  of the real axis (b) line segment  $[-2, 2]$  of the imaginary axis  
 (c) Unit circle  $|z|=1$  (d) the line with  $\arg z = \tan^{-1}2$

**SOLUTION : (A)** assuming  $\arg z_1 = \theta$  and  $\arg z_2 = \theta + \alpha$ ,

$$\frac{az_1}{bz_2} + \frac{bz_2}{az_1} = \frac{a|z_1|e^{i\theta}}{b|z_2|e^{i(\theta+\alpha)}} + \frac{b|z_2|e^{i(\theta+\alpha)}}{a|z_1|e^{i\theta}}$$

$$= e^{i\alpha} + e^{-i\alpha} = 2 \cos \alpha$$

Hence, the point lies on the line segment  $[-2, 2]$  of the real axis.

12. If  $ax^3 + bx^2 + cx + d$  is divisible by  $ax^2 + c$ , then  $a, b, c, d$  are in  
 (a) A.P. (b) G.P. (c) H.P. (d) none of these

**SOLUTION : (D)** since  $ax^3 + bx^2 + cx + d$  is divisible by  $ax^2 + c$ , when  $ax^3 + c$  the remainder should be zero. if  $ax^3 + bx^2 + cx + d$  is divided by  $ax^2 + c$ , then the remainder is  $(bc/a) - d$ . therefore,

$$\frac{bc}{a} - d = 0$$

$$\Rightarrow bc = ad$$

$$\Rightarrow \frac{b}{a} = \frac{d}{c}$$

Hence, from this,  $a, b, c, d$  are not necessarily in G.P.

13. The number of ordered pairs of integer  $(x, y)$  satisfying the equation  $x^2 + 6x + y^2 = 4$  is  
 (a) 2 (b) 8 (c) 6 (d) none of these

**SOLUTION : (B).**  $(x + 3)^2 + y^2 = 13$

$$\Rightarrow x + 3 = \pm 2, y = \pm 3 \text{ or } x + 3 = \pm 3, y = \pm 2$$

14. If  $f(3x + 2) + f(3x + 29) = 0 \forall x \in R$ , then the period of  $f(x)$  is  
 (a) 7 (b) 8 (c) 10 (d) none of these

**SOLUTION : (D)**  $f(3x + 2) + f(3x + 29) = 0$  (1)

Replacing  $x$  by  $x + 9$ , we get

$$f(3(x + 9) + 2) + f(3(x + 9) + 29) = 0$$

$$\Rightarrow f(3x + 29) + f(3x + 56) = 0$$
 (2)

From (1) and (2), we get

$$f(3x + 2) = f(3x + 56)$$

$$\Rightarrow f(3x + 2) = f(3(x + 18) + 2)$$

$\Rightarrow f(x)$  is periodic with period 18.

15. If  $(\cos \theta + i \sin \theta)(\cos 2\theta + i \sin 2\theta) \dots (\cos n\theta + i \sin n\theta) = 1$ , then the value of  $\theta$  is:

(a)  $4m\pi$  (b)  $\frac{2m\pi}{n(n+1)}$  (c)  $\frac{4m\pi}{n(n+1)}$  (d)  $\frac{m\pi}{n(n+1)}$

**Solution(c)** we have  $(\cos \theta + i \sin \theta)(\cos 2\theta + i \sin 2\theta) \dots (\cos n\theta + i \sin n\theta) = 1$

$$\Rightarrow \cos(\theta + 2\theta + 3\theta + \dots + n\theta) + i \sin(\theta + 2\theta + \dots + n\theta) = 1$$

$$\Rightarrow \cos\left(\frac{n(n+1)}{2}\theta\right) + i \sin\left(\frac{n(n+1)}{2}\theta\right) = 1$$

$$\cos\left(\frac{n(n+1)}{2}\theta\right) = 1 \text{ and } \sin\left(\frac{n(n+1)}{2}\theta\right) = 0$$

$$\Rightarrow \frac{n(n+1)}{2}\theta = 2m\pi$$

$$\Rightarrow \theta = \frac{4m\pi}{n(n+1)}, \text{ where } m \in I.$$

16. If  $(1 + i)(1 + 2i)(1 + 3i) \dots (1 + ni) = a + ib$ , then  $2.5.10 \dots (1 + n^2)$  is equal to:

(a)  $a^2 - b^2$  (b)  $a^2 + b^2$  (c)  $\sqrt{a^2 + b^2}$  (d)  $\sqrt{a^2 - b^2}$

**Solution:** (b) we have  $(1 + i)(1 + 2i)(1 + 3i) \dots (1 + ni) = a + ib$  ... (i)

$$\Rightarrow (1 - i)(1 - 2i)(1 - 3i) \dots (1 - ni) = a - ib$$
 ... (ii)

Multiplying Eqs. (i) and (ii),

$$\text{We get } 2.5 \dots (1 + n^2) = a^2 + b^2$$

17. Let  $z_1$  and  $z_2$  be  $n$ th roots of unity which are ends of a line segment that subtend a right angle at the origin. Then  $n$  must be of the form:

(a)  $4k + 1$  (b)  $4k + 2$  (c)  $4k + 3$  (d)  $4k$

**Solution:** (d)  $1^{1/n} = \cos \frac{2r_1\pi}{n} + i \sin \frac{2r_1\pi}{n}$

$$\text{Let } z_1 = \cos \frac{2r_1\pi}{n} + i \sin \frac{2r_1\pi}{n}$$

$$\text{And } z_2 = \cos \frac{2r_2\pi}{n} + i \sin \frac{2r_2\pi}{n}$$

$$\text{Then } \angle z_1 O z_2 = \text{amp} \left( \frac{z_1}{z_2} \right) = \text{amp} (z_1) - \text{amp} (z_2) = \frac{2(r_1 - r_2)\pi}{n} = \frac{\pi}{2} \text{ (Given)}$$

$$\therefore n = 4(r_1 - r_2) = 4 \times \text{integer, so } n \text{ is of the form } 4k.$$

18. Let  $z$  and  $\omega$  be the two non-zero complex numbers such that  $|z| = |\omega|$  and  $\arg z + \arg \omega = \pi$ . Then  $z$  is equal to:

(a)  $\omega$  (b)  $-\omega$  (c)  $\bar{\omega}$  (d)  $-\bar{\omega}$

**Solution:** (d)  $\because |z| = |\omega| \Rightarrow |z| = |\bar{\omega}|$

$$\arg z + \arg \omega = \pi \Rightarrow \arg z - \arg(\bar{\omega}) = \pi$$

$$\therefore z + \bar{\omega} = 0 \Rightarrow z = -\bar{\omega}$$

19. If  $z$  and  $\omega$  are two non-zero complex numbers such that  $|z\omega| = 1$  and  $\arg(z) - \arg(\omega) = \frac{\pi}{2}$ , then  $\bar{z}\omega$  is equal to:

- (a) 1                      (b) -1                      (c) i                      (d) -i

**Solution**(d)  $|z||\omega| = 1$  (i)

And  $\arg\left(\frac{z}{\omega}\right) = \frac{\pi}{2} \Rightarrow \frac{z}{\omega} = i \Rightarrow \left|\frac{z}{\omega}\right| = 1$  (ii)  
From equations (i) and (ii)

$$|z| = |\omega| = 1 \text{ and } \frac{z}{\omega} + \frac{\bar{z}}{\bar{\omega}} = 0; z\bar{\omega} + \bar{z}\omega = -i$$

20. If  $\left(\frac{1+i}{1-i}\right)^x = 1$ , then:

- (a)  $x = 4n$ , where  $n$  is any positive integer  
(b)  $x = 2n$ , where  $n$  is any positive integer  
(c)  $x = 4n + 1$ , where  $n$  is any positive integer  
(d)  $x = 2n + 1$ , where  $n$  is any positive integer

**Solution:** (a)  $\left(\frac{1+i}{1-i}\right)^x = 1 \Rightarrow \left[\frac{(1+i)^2}{(1-i)^2}\right]^x = 1$

$$\Rightarrow \left(\frac{1+i^2+2i}{1+i^2}\right)^x = 1 \Rightarrow i^x = 1$$

$$\therefore x = 4n, n \in I^+$$

21. If  $z_1, z_2, z_3$  are complex numbers such that  $|z_1| = |z_2| = |z_3| = \left|\frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3}\right| = 1$ , then  $|z_1 + z_2 + z_3|$  is:

- (a) Equals to 1    (b) Less than 1    (c) Greater than 3    (d) Equal to 3

**Solution:** (a)  $1 = \left|\frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3}\right| = \left|\frac{z_2\bar{z}_1 + z_3\bar{z}_2 + z_1\bar{z}_3}{z_1z_2z_3}\right|$  ( $\because |z_1|^2 = 1 = z_1\bar{z}_1$ , etc)

$$= \left|\frac{\bar{z}_1 + \bar{z}_2 + \bar{z}_3}{z_1z_2z_3}\right| = \left|\bar{z}_1 + \bar{z}_2 + \bar{z}_3\right| = |z_1 + z_2 + z_3| \quad (\because |\bar{z}_1| = |z_1|)$$

22. Let  $z, \omega$  be complex numbers such that  $z + i\bar{\omega} = 0$  and  $\arg z\omega = \pi$ , then  $\arg z$  equals:

- (a)  $\frac{5\pi}{4}$                       (b)  $\frac{\pi}{2}$                       (c)  $\frac{3\pi}{4}$                       (d)  $\frac{\pi}{4}$

**Solution:** (c) Given that  $\arg z\omega = \pi$  (i)

$$\bar{z} + i\bar{\omega} = 0 \Rightarrow \bar{z} = -i\bar{\omega}$$

$$\Rightarrow z = i\omega \Rightarrow \omega = -iz$$

From Eq. (i),  $\arg(-iz^2) = \pi$

$$\text{Arg}(-i) + 2\arg(z) = \pi; \frac{-\pi}{2} + 2\arg(z) = \pi$$

$$2\arg(z) = \frac{3\pi}{2}; \arg(z) = \frac{3\pi}{4}$$

23. The value of  $\sum_{k=1}^6 \left(\sin \frac{2\pi k}{7} - i \cos \frac{2\pi k}{7}\right)$  is:

- (a) -1                      (b) 0                      (c) -i                      (d) i

**Solution:** (d)  $\sum_{k=1}^6 \left(\sin \left(\frac{2\pi k}{7}\right) - i \cos \left(\frac{2\pi k}{7}\right)\right)$

$$= -i \sum_{k=1}^6 \left(\sin \frac{2\pi k}{7} + i \cos \frac{2\pi k}{7}\right)$$

$$= -i \sum_{k=1}^6 e^{\frac{2\pi k}{7}}$$

$$= -i \left(\sum_{k=1}^6 e^{\frac{2\pi k}{7}} - 1\right)$$

$$= -i(\text{Sum of 7, 7th roots of unity} - 1)$$

$$= -i(0 - 1) = i$$

24. If  $z_1$  and  $z_2$  are two non-zero complex numbers such that  $|z_1 + z_2| = |z_1| + |z_2|$ , then  $\arg(z_1) - \arg(z_2)$  is equal to:

- (a)  $-\pi$                       (b)  $-\frac{\pi}{2}$                       (c)  $\frac{\pi}{2}$                       (d) 0

**Solution:** (d) Let  $z_1 = r_1(\cos \theta_1 + i \sin \theta_1)$ ,  $z_2 = r_2(\cos \theta_2 + i \sin \theta_2)$

$$\therefore |z_1 + z_2| = [(r_1 \cos \theta_1 + r_2 \cos \theta_2)^2 + (r_1 \sin \theta_1 + r_2 \sin \theta_2)^2]^{1/2}$$

$$= |r_1^2 + r_2^2 + 2r_1r_2 \cos(\theta_1 - \theta_2)|^{1/2}$$

$$= [(r_1 + r_2)^2]^{1/2}$$

$$\therefore |z_1 + z_2| = |z_1| + |z_2|$$

$$\text{Therefore, } \cos(\theta_1 - \theta_2) = 1$$

$$\Rightarrow \theta_1 - \theta_2 = 0 \Rightarrow \theta_1 = \theta_2$$

$$\text{Thus, } \arg(z_1) - \arg(z_2) = 0$$

25. If  $\omega = \alpha + i\beta$ , where  $\beta \neq 0$  and  $z \neq 1$ , satisfies the condition that  $\left(\frac{\omega - \bar{\omega}z}{1-z}\right)$  is purely real, then the set of value of  $z$  is:

- (a)  $\{z: |z| = 1\}$     (b)  $\{z: z = \bar{z}\}$     (c)  $\{z: z \neq 1\}$     (d)  $\{z: \bar{z} = 1, z \neq 1\}$

**Solution:** (d) Given  $\left(\frac{\omega - \bar{\omega}z}{1-z}\right)$  is purely real  $\Rightarrow z \neq 1$

$$\therefore \left(\frac{\omega - \bar{\omega}z}{1-z}\right) = \overline{\left(\frac{\omega - \bar{\omega}z}{1-z}\right)}$$

$$= \frac{\bar{\omega} - \omega\bar{z}}{1-\bar{z}}$$

$$\Rightarrow (\omega - \bar{\omega}z)(1 - \bar{z}) = (1 - z)(\bar{\omega} - \omega\bar{z})$$

$$\Rightarrow (z\bar{z} - 1)(\omega - \bar{\omega}) = 0$$

$$\Rightarrow (|z|^2 - 1)(2i\beta) = 0 \quad (\because \omega = \alpha + i\beta)$$

$$\therefore |z|^2 - 1 = 0$$

$$\Rightarrow |z| = 1 \text{ and } z \neq 1 \quad (\beta \neq 0)$$

26. A man walks a distance of 3 units from the origin towards the north-east ( $N 45^\circ E$ ) direction. From there, he walks a distance of 4 units towards the north-west ( $N 45^\circ W$ ) direction to reach a point P. then the position of P in the Argand Plane is:

- (a)  $3e^{i\pi/4} + 4i$     (b)  $(3 - 4i)e^{i\pi/4}$     (c)  $(4 + 3i)e^{i\pi/4}$     (d)  $(3 + 4i)e^{i\pi/4}$

**Solution:** (d) Let  $OA = 3$ , so that the complex number associated with A is  $3e^{i\pi/4}$ . If  $z$  is the complex number associated with P, then

$$\frac{z - 3e^{i\pi/4}}{0 - 3e^{i\pi/4}} = \frac{4}{3} e^{i\pi/4} = -\frac{4i}{3}$$

$$\Rightarrow 3z - 9e^{i\pi/4} = 12ie^{i\pi/4}$$

$$\Rightarrow z = (3 + 4i)e^{i\pi/4}$$

27. If the quadratic equation

$$z^2 + (a + ib)z + c + id = 0$$

Where  $a, b, c, d$  are non-zero real numbers has a real root, then

(a)  $abd = b^2c + d^2$

(b)  $abd = bc^2 + d^2$

(c)  $abd = bc^2 + ad^2$

(d) none of these

**Solution :** (a) Let the real number  $a$  be a root of  $z^2 + (a + ib)z + c + id = 0$

$$\Rightarrow a^2 + (a + ib)a + c + id = 0$$

$$\Rightarrow a^2 + aa + c = 0 \text{ and } ba + d = 0$$

Eliminating  $a$  we obtain

$$\left(-\frac{d}{b}\right)^2 + a\left(-\frac{d}{b}\right) + c = 0$$

$$\Rightarrow d^2 + abd + b^2c = 0 \Rightarrow abd = bc^2 + d^2$$

28. The locus of  $z$ , which satisfies the inequality :

$$\log_{0.3}|z = 1| > \log_{0.3}|z - i|$$
 is given by :

(a)  $x + y < 0$

(b)  $x - y > 0$

(c)  $x + y > 0$

(d)  $x - y < 0$

**Solution : (b)** By the question,  $|z = 1| < |z - i|$

$$\Rightarrow |x + iy - 1| < |x + iy - i|$$

$$\Rightarrow |(x - 1) + iy| < |x + i(y - 1)|$$

$$\Rightarrow \sqrt{(x - 1)^2 + y^2} < \sqrt{x^2 + (y - 1)^2}$$

$$\Rightarrow (x - 1)^2 + y^2 < x^2 + (y - 1)^2$$

$$\Rightarrow x^2 - 2x + 1 + y^2 < x^2 + y^2 - 2y + 1$$

$$\Rightarrow -2x < -2y \Rightarrow -x < -y$$

$$\Rightarrow 0 < x - y \Rightarrow x - y > 0.$$

29. If  $\log_{\sqrt{3}} 5 = a$  and  $\log_{\sqrt{3}} 2 = b$ , then  $\log_{\sqrt{3}} 300 =$

[a]  $2(a + b)$                       [b]  $2(a + b + 1)$                       [c]  $2(a + b + 2)$                       [d]  $a + b + 4$

**Solution :- [c]**

$$\log_{\sqrt{3}} 300 = \log_{\sqrt{3}} (3 \times 2^2 \times 5^2)$$

$$= \log_{\sqrt{3}} 3 + 2 \log_{\sqrt{3}} 2 + 2 \log_{\sqrt{3}} 5$$

$$= 2 \log_3 3 + 2b + 2a$$

$$(\because \log_{\sqrt{3}} 5 = a \text{ and } \log_{\sqrt{3}} 2 = b)$$

$$= 2(a + b + 1)$$

30. The function  $f(x) = \sin \left( \log \left( x + \sqrt{x^2 + 1} \right) \right)$  is :

[a] Even function

[b] odd function

[c] Neither even nor odd

[d] Periodic function

**Solution :- [b]**

$$f(x) = \sin(\log(x + \sqrt{1 + x^2}))$$

$$\Rightarrow f(-x) = \sin[\log(-x + \sqrt{1 + x^2})]$$

$$\Rightarrow f(-x) = \sin \log \left( (\sqrt{1 + x^2} - x) \frac{(\sqrt{1 + x^2} + x)}{(\sqrt{1 + x^2} + x)} \right)$$

$$\Rightarrow f(-x) = \sin \log \left[ \frac{1}{(x + \sqrt{1 + x^2})} \right]$$

$$\Rightarrow f(-x) = \sin \left[ \log(x + \sqrt{1 + x^2})^{-1} \right]$$

$$\Rightarrow f(-x) = \sin[-\log(x + \sqrt{1 + x^2})]$$

$$\Rightarrow f(-x) = -\sin[\log(x + \sqrt{1 + x^2})]$$

$$\Rightarrow f(-x) = -f(x)$$

$\therefore f(x)$  is odd function.