Mathematics Question paper with Solutions

Com. **Class – XII**

1. The set A has 4 elements and the set B has 5 elements, then the number of injective mappings that can be defined from A to B is

   (a) 144 (b) 72 (c) 60 (d) 120

Ans. D, Total number of injective mappings from set A to set B = ${}^5P_4 = \frac{5!}{1!} = 120$

2. Let f : R → R be defined by f(x) = 2x + 6, which is a objective mapping, then f^{-1}(x) is given by

   (a) $\frac{\pi}{2} - 3$ (b) 2x + 6 (c) x – 3 (d) 6x + 2

Ans. A, We have, $f(x) = 2x + 6$

We have that, $f\{f^{-1}(x)\} = x$

∴ $2f^{-1}(x) + 6 = x$

⟹ $2f^{-1}(x) = x - 6$

⟹ $f^{-1}(x) = \frac{x-6}{2}$

⟹ $f^{-1}(x) = \frac{x}{2} - 3$

3. Let * be a binary operation defined on R by a * b= $\frac{a+b}{4}$, $\forall a, b \in R$, then the operation defined on R by a * b = $\frac{a+b}{4}$, $\forall a, b \in R$, then the operation * is

   (a) Commutative and associative

   (b) Commutative but not associative

   (c) Associative but not commutative

   (d) Neither associative nor commutative

Ans. B, We have, a * b = $\frac{a+b}{4}$

For commutative, a * b = b * a

∴ $\frac{a+b}{4} = \frac{b+a}{4}$

Hence, * is commutative,

For association, a * (b * c) = (a * b) * c

LHS = a * $\left(\frac{b+c}{4}\right) = \frac{a+\frac{b+c}{4}}{4} = \frac{4a+b+c}{16}$

RHS = (a * b) * c = $\left(\frac{a+b}{4}\right) * c$

$= \frac{\frac{a+b}{4}+c}{4} = \frac{a+b+4c}{16}$

Since, a * (b * c) ≠ (a *b) * c

So, * is not associative.

4. The value of $sin^{-1}\left(\cos\frac{53\pi}{5}\right)$ *is*

   (a) $\frac{3\pi}{5}$ (b) $\frac{-3\pi}{5}$ (c) $\frac{\pi}{10}$ (d) $\frac{-\pi}{10}$

Ans. D, We have,

$sin^{-1}\left(\cos\frac{53\pi}{5}\right) = sin^{-1}\left[\cos\left(10\pi + \frac{3\pi}{5}\right)\right]$

$= sin^{-1}\left[\cos\left(\frac{3\pi}{5}\right)\right]$ [∵ cos $(2n\pi \pm \theta)$ = cos θ]

$= sin^{-1}\left[\sin\left(\frac{\pi}{2} - \frac{3\pi}{5}\right)\right]$

$= sin^{-1}\left[\sin\left(\frac{5\pi-6\pi}{10}\right)\right]$

$= sin^{-1}\left[\sin\left(\frac{-\pi}{10}\right)\right] = \frac{-\pi}{10}$

5. If 3 tan^{-1} x + cot^{-1} x = π, then is equal to

(a) 0 (b) 1 (c) -1 (d) ½

Ans. B, we have, 3 $tan^{-1} x + cot^{-1} x = \pi$

$\Longrightarrow 2\ tan^{-1} x + tan^{-1} x + cot^{-1} x = \pi$

$\Longrightarrow$ $2\ tan^{-1}$ x + $\frac{\pi}{2}$ = π $\left[\because tan^{-1} x + cot^{-1} x = \frac{\pi}{2}\right]$

$\Longrightarrow$ $2tan^{-1}$ x = $\pi - \frac{\pi}{2}$

$\Longrightarrow$ $tan^{-1} x = \frac{\pi}{4} \Longrightarrow$ x = tan $\frac{\pi}{4}$

$\therefore$ x = 1

6. The simplified form of $tan^{-1}\left(\frac{x}{y}\right) - tan^{-1}\left(\frac{x-y}{x+y}\right)$ *is equal to*

(a) 0 (b)$\pi/4$ (c)$\pi/2$ (d)π

Ans. B, we have, $tan^{-1}\left(\frac{x}{y}\right) - tan^{-1}\left(\frac{x-y}{x+y}\right)$

$= tan^{-1}\left(\frac{x}{y}\right) + tan^{-1}\left(\frac{y-x}{y+x}\right)$

$= tan^{-1}\left(\frac{x}{y}\right) + tan^{-1}\left(\frac{1-x/y}{1+x/y}\right)$

$= tan^{-1}\left(\frac{x}{y}\right) + tan^{-1}(1) - tan^{-1}\left(\frac{x}{y}\right) = tan^{-1}(1)$

$\left[\because tan^{-1} A - tan^{-1} B = tan^{-1}\left(\frac{A-B}{1+AB}\right)\right]$

$=\frac{\pi}{4}$

7. If x, y, z are all different and not equal to zero and $\begin{vmatrix} 1+x & 1 & 1 \\ 1 & 1+y & 1 \\ 1 & 1 & 1+z \end{vmatrix}$=0, then the value of $x^{-1} + y^{-1} + z^{-1}$ is equal to

(a) Xyz (b) $x^{-1}y^{-1}z^{-1}$ (c) – x – y – z (d) -1

Ans. D, Given, $\begin{vmatrix} 1+x & 1 & 1 \\ 1 & 1+y & 1 \\ 1 & 1 & 1+z \end{vmatrix}$=0

Expanding along R_1, we get

(1 + z)[(1 + y)(1 + z)-1] – 1(1 + z - 1) + 1(1 – 1 – y) = 0

$\Longrightarrow$ $(1+x)(1+y)(1+z) = x + y + z + 1$

$\Longrightarrow$ 1 + x + y +z + xy + yz + xyz = x + y + z + 1

$\Longrightarrow$ xy + yz + xz = - xyz

On dividing both sides by xyz, we get

$\frac{1}{z} + \frac{1}{x} + \frac{1}{y} = -1 \Longrightarrow x^{-1} + y^{-1} + z^{-1}$ = - 1

8. If y = $e^{sin^{-1}(t^2-1)}$ *and* $x = e^{sec^{-1}\left(\frac{1}{t^2-1}\right)}$, *then* $\frac{dy}{dx}$ *is equal to*

(a) $\frac{x}{y}$ (b) $\frac{-y}{x}$ (c) $\frac{y}{x}$ (d) $\frac{-x}{y}$

Ans. B, we have, y = $e^{sin^{-1}(t^2-1)}$ and x = $e^{sec^{-1}\left(\frac{1}{t^2-1}\right)}$

Now, xy = $e^{sin^{-1}(t^2-1)}$. $e^{sec^{-1}\left(\frac{1}{t^2-1}\right)}$

$\Longrightarrow$ xy = $e^{sin^{-1}(t^2-1)+ cos^{-1}(t^2-1)}$

$$\left[\because sec^{-1}\left(\frac{1}{t^2-1}\right) = cos^{-1}(t^2-1)\right]$$

$\Longrightarrow$ xy = $e^{\pi/2}$ $\left[\because sin^{-1}\theta + cos^{-1}\theta = \frac{\pi}{2}\right]$

On differentiating both sides w.r.t. x, we get

$$x\frac{dy}{dx} + y = 0$$

$$\Longrightarrow \quad \frac{dy}{dx} = -\frac{y}{x}$$

9. If A = $\frac{1}{\pi}\begin{bmatrix} sin^{-1}(\pi x) & tan^{-1}\left(\frac{x}{\pi}\right) \\ sin^{-1}\left(\frac{\pi}{x}\right) & cot^{-1}(\pi x)\end{bmatrix}$, B = $\frac{1}{\pi}\begin{bmatrix} -cos^{-1}(\pi x) & tan^{-1}\left(\frac{x}{\pi}\right) \\ sin^{-1}\left(\frac{\pi}{x}\right) & -tan^{-1}(\pi x)\end{bmatrix}$, then A – B is equal to

(a) I (b) 0 (c) 2I (d) $\frac{1}{2}I$

Ans. D, we have, A = $\frac{1}{\pi}\begin{bmatrix} sin^{-1}(\pi x) & tan^{-1}\left(\frac{x}{\pi}\right) \\ sin^{-1}\left(\frac{x}{\pi}\right) & cot^{-1}(\pi x)\end{bmatrix}$

And B = $\frac{1}{\pi}\begin{bmatrix} -cos^{-1}(\pi r) & tan^{-1}\left(\frac{x}{\pi}\right) \\ sin^{-1}\left(\frac{x}{\pi}\right) & -tan^{-1}(\pi x)\end{bmatrix}$

∴ A – B

$$=\frac{1}{\pi}\begin{bmatrix} sin^{-1}(\pi x) + cos^{-1}(\pi x) & tan^{-1}\left(\frac{x}{\pi}\right) - tan^{-1}\left(\frac{x}{\pi}\right) \\ sin^{-1}\left(\frac{x}{\pi}\right) - sin^{-1}\left(\frac{x}{\pi}\right) & cot^{-1}(\pi x) + tan^{-1}(\pi x)\end{bmatrix}$$

$$=\frac{1}{\pi}\begin{bmatrix}\frac{\pi}{2} & 0 \\ 0 & \frac{\pi}{2}\end{bmatrix} \qquad \begin{bmatrix}\because sin^{-1}x + cos^{-1}x = \frac{\pi}{2} \\ and\ cot^{-1}x + tan^{-1}x = \frac{\pi}{2}\end{bmatrix}$$

$$=\begin{bmatrix}\frac{1}{2} & 0 \\ 0 & \frac{1}{2}\end{bmatrix} = \frac{1}{2}\begin{bmatrix}1 & 0 \\ 0 & 1\end{bmatrix} = \frac{1}{2}I$$

10. The value of $\int \frac{e^x(1+x)}{cos^2(e^x.\ x)}$ dx is equal to

(a) – cot (ex^x)+ C (b) tan($e^x\ .x$)+ C (c) tan (e^x) + C (d) cot(e^x) + C

Ans. B, Let $I = \int \frac{e^x\,(1+x)}{cos^2(e^x\,.x)}\ dx$

Put $e^x\,.x = t$

$\Longrightarrow$ ($e^x + xe^x$) dx = dt

$\Longrightarrow$ e^x(x + 1) dx = dt

$$\therefore \quad I = \int\frac{dt}{cos^2 t} = \int sec^2 t\ dt = \tan t + c$$

= tan($e^x\ .x$) + C

11. The value of $\int \frac{e^x\,(x^2\,tan^{-1}x + tan^{-1}x + 1)}{x^2+1}\,dx$ is equal to

(a) $e^x\,tan^{-1}x + C$ (b) $tan^{-1}(e^x) + c$ (c) $tan^{-1}(x^e) + C$ (d) $e^{tan^{-1}}(e^x) + C$

Ans. A, Let $I = \int e^x\left(\frac{x^2\,tan^{-1}x + tan^{-1}x + 1}{x^2+1}\right)dx$

$\Longrightarrow$ $I = \int e^x\left(tan^{-1}x + \frac{1}{x^2+1}\right)dx$

If $f(x) = tan^{-1}x, then$

$f'(x) = \frac{1}{x^2+1}$

$\therefore$ $I = f e^x\left(tan^{-1}x + \frac{1}{x^2+1}\right)\,dx$

$= e^2\,tan^{-1}x + C$

$[\because\ \int e^2\,[f(x) + f'(x)]\,dx = e^x\,f(x) + C]$

12. The differential coefficient of log_{10} x with respect to log_x 10 is

(a) 11 (b) $-(log_{10}x)^2$ (c) $(log_x\,10)^2$ (d) $\frac{x^2}{100}$

Ans. B, Let $= log_{10}\,x\ and\ v = log_x 10$

$\Longrightarrow$ u = $\frac{log_e\,x}{log_e\,10}$ and v = $\frac{log_e 10}{log_e\,x}$

Now, $\frac{du}{dx} = \frac{1}{x\,log_e\,10}$

And $\frac{du}{dx} = log_e 10\,\left(\frac{-1}{x(log_e\,x)^2}\right)$

$\therefore$ $\frac{du}{dv} = \frac{du/dx}{dv/dx} = \frac{1}{x\,log_{e\,10}} + \frac{-\,log_e\,10}{x(log_e\,x)^2}$

$= \frac{-(log_e\,x)^2}{(log_e\,10)^2} = -\left(\frac{log_e\,x}{log_{e\,10}}\right)^2$

$= -(log_{10}\,x)^2$

13. $\int_0^{\frac{\pi}{2}} \frac{sin^{1000}\,x}{sin^{1000}\,x + cos^{1000}x}\ dx\ is\ equal\ to$

(a) 1000 (b) 1 (c)$\frac{\pi}{2}$ (d)$\frac{\pi}{4}$

Ans. D, let $I = \int_0^{\pi/2} \frac{sin^{1000}\,x}{sin^{1000},x + cos^{1000}\,x}\ dx$ ...(i)

$\Longrightarrow$ $I = \int_0^{\pi/2} \frac{sin^{1000}\left(\frac{\pi}{2} - x\right)}{sin^{1000}\left(\frac{\pi}{2} - \pi\right) + cos^{1000}\left(\frac{\pi}{2} - x\right)}\,dx$

$[\because\ \int_0^a f(x)dx = \int_0^a f(a-x)dx]$

$\Longrightarrow$ $I = \int_0^{\pi/2} \frac{cos^{1000}\,x}{cos^{1000}\,x + sin^{1000}\,x}\ dx$ ...(iii)

On adding Eqs. (i) and (ii), we get

$2I = \int_0^{\pi/2} \frac{sin^{1000}\,x + cos^{1000}\,x}{sin^{1000}\,x + cos^{1000}x}\ dx$

$\Longrightarrow$ $2I = \int_0^{\pi/2} 1\ dx = [x]_0^{\pi/2} \Longrightarrow 2I = \frac{\pi}{2}$

$\therefore$ $I = \frac{\pi}{4}$

14. If the function $f(x)$ and g(x) are continuous on[a, b] and differentiable on (a, b), then in the interval (a, b), the equation $\begin{vmatrix} f'(x) & f(a) \\ g'(x) & f(a) \end{vmatrix} = \frac{1}{a-b}\begin{vmatrix} f(a) & f(b) \\ g(a) & g(b) \end{vmatrix}$ has

(a) atleast one root (b) exactly one root (c) atmost one root (d) no root

Ans. A, Consider the function $\Phi(x)$ given by

$$\Phi(x) = \begin{vmatrix} f(a) & f(x) \\ g(a) & g(x) \end{vmatrix}$$

Since, $f(x)$ and g(x) are continuous on [a, b] and differentiable on (a, b). Therefore, $\Phi(x)$ is continuous on [a, b] and differentiable on(a, b). consequently, by Langrange's mean value theorem, there exists atleast one point c∈ (a, b) such that

$$\Phi'(c) = \frac{\Phi(b)-\Phi(a)}{b-a}$$

$$\Rightarrow \quad \begin{vmatrix} f(a) & f'(c) \\ g(a) & g'(c) \end{vmatrix} = \frac{1}{b-a}\begin{vmatrix} f(a) & f(b) \\ g(a) & g(b) \end{vmatrix}$$

$$\Rightarrow \quad \begin{vmatrix} f'(a) & f(c) \\ g'(a) & g(c) \end{vmatrix} = \frac{1}{b-a}\begin{vmatrix} f(a) & f(b) \\ g(a) & g(b) \end{vmatrix}$$

Hence, the equation

$$\begin{vmatrix} f'(x) & f(c) \\ g'(x) & g(c) \end{vmatrix} = \frac{1}{b-a}\begin{vmatrix} f(a) & f(b) \\ g(a) & g(b) \end{vmatrix}$$

Has atleast one root in (a, b).

15. If a function is everywhere continuous and differentiable such that f'(x) ≥ 6 for all x ∈ [2, 4] and $f(2)$ = - 4, then

(a) $f(4) < 8$ (b) $f(4) \geq 8$ (c) $f(4) \geq 2$ (d) $f(4) \leq 2$

Ans. B, Since $f(x)$ is everywhere continuous and differentiable. Therefore, by Lagrange's mean value theorem, there exits c ∈ $(2,4)$ such that

$$f'(c) = \frac{f(4)-f(2)}{4-2} \Rightarrow \frac{f(4)+4}{2} \geq 6$$

$$[\because f'(x) \geq 6 \; for \; all \; x \; \in [24] and \; f(2) = -4]$$

$$\Rightarrow \quad f(4) \geq 8$$

16. In $[0,1]$, $Langrage's$ means value theorem is not applicable to

(a) $f(x) = \begin{cases} \frac{1}{2} - x, \; x < \frac{1}{2} \\ \left(\frac{1}{2} - x\right)^2, x \geq \frac{1}{2} \end{cases}$

(b) $f(x) = \begin{cases} \frac{\sin x}{x} \;, \; x \neq 0 \\ 1 \;, \; x = 0 \end{cases}$

(c) $f(x)$ = x |x|

(d) $f(x) = |x|$

Ans. A, For the function $f(x)$ given in option (a), we have

$$\left(LHD \; at \; x = \frac{1}{2}\right) = -1$$

And $$\left(RHD \; at \; x = \frac{1}{2}\right) = 0$$

So, it is not differentiable at x = $\frac{1}{2}$ ∈ $(0,1)$.

Hence, Lagrange's mean value theorem is not applicable.

17. If Rolle's theorem hold for the function $f(x) = x^3 + bx \; cx, 1 \leq x \leq 2$ at the point 4/3, then the values of b and c are

(a) b = 8, c = -5 (b) b = -5, c = 8 (c) b = 5, c = - 8 (d) b = - 5, c = -8

Ans. B, It is given that Rolle's theorem holds for $f(x) = x^3 + bx^2 + cx$ on[1, 2].

∴ $f(1) = f(2)$

And $f'\left(\frac{4}{3}\right) = 0$

$\Longrightarrow$ 1 + b + c = 8 + 4b + 2c

And $3\left(\frac{4}{3}\right)^2 + 2b\left(\frac{4}{3}\right) + c = 0$ $[\because f'(x) = 3\,x^2 + 2bx + c]$

$\Longrightarrow$ 3b + c + 7 = 0

And 8b + 3c + 16 = 0

$\Longrightarrow$ b= -5 and c = 8

18. let $f(x) satisfy\ the$ requirement of Lagrange's mean value theorem in [0, 2]. If $f(0)$ = 0 and $|f'$ (x)$| \leq \frac{1}{2}$ for all x ∈ [0, 2], then

(a) $f(x) \leq 2$ (b) $|f(x)| \leq 1$ (c) $f(x) = 2x$ (d) $f(x) = 3$ for at least one x in [0,2]

Ans. B, Let x ∈ (0, 2). Since, $f(x)$ satisfies the requirement of langrage's mean value theorem in $[0, 2]$. So, it also satisfies in$[0, x]$. Consequently, there exists c∈(0, x), such that

$$f'(c) = \frac{f(x)-f(0)}{x-0}$$

$\Longrightarrow$ $f'(c) = \frac{f(x)}{x}$

$\Longrightarrow$ $\left|\frac{f'(x)}{x}\right| = |f'(x)| \leq \frac{1}{2}$ $\left[\because |f'(x)| \leq \frac{1}{2}\right]$

$\Longrightarrow$ $|f(x)| \leq \frac{|x|}{2}$

$|f(x)| \leq \frac{|x|}{2}$ $[\because x \ \geq \ 0]$

$\Longrightarrow$ $|f(x)| \leq 1$ $[\because x \ \in (0, 2) \ \Longrightarrow |x| < 2]$

19. Let f: [0, 4] → R be a differentiable function. Then, there exists real numbers a, b belonging to (0,4) such that $[f(4)]^2$ - $[f(0)]^2$ = kf'(a) $f(b)$, where k is

(a) 4 (b) 8 (c) $\frac{1}{12}$ (d) 2

Ans. B, By Lagrange's mean value theorem, there exists a ∈ (0, 4) such that

$$\frac{f(4)-f(0)}{4-0} = f'(a)$$

Therefore, $\frac{f(4)-f(0)}{4-0} = 4f'(a)$ ...(i)

Since,$[f(4) + f(0)]/2$ lies between $f(0) and\ f(4)$, by the intermediate value property of a continuous functions, there exists b ∈ (0, 4) such that

$$\frac{f(4)+f(0)}{2} = f'(b)$$

$\Longrightarrow$ $f(4) + f(0) = 2f(b)$ ...(ii)

From Eqs. (i) and (ii), we get

$$[f(4)]^2 - [f(0)]^2 = 8f'(a)f(b) \Longrightarrow k = 8$$

20. f is twice differentiable function on[a, b] such that $f(a) = f(b)$ = 0 for all x ∈(a, b). Then,

(a) f''(x) > 0, ∀ x ∈ (a, b)

(b) f''(x) > 0, ∀ x ∈ (a, b)

(c) $f''(x_0) < 0$ for some $x_0 \in (c_{1,} c_2)$

(d) $f''(x_0) = 0$ for some $x_0 \in (c_{1,} c_2)$

Ans. C, Using Rolle's theorem for f on $[a, b]$, there exists c ∈ (a, b) such that f'(c) = 0. Now, using Langrange's mean value theorem for f on the intervals $[a, c]$ and $[c, b]$, there exist c_1 ∈ (a, c) and c_2 ∈ (c, b) such that

$\frac{f(c)-f(a)}{c-a} = f'(c_1)$ and $\frac{f(b)- f(c)}{b-c} = f'(c_2)$

Now, use Langrage's mean value theorem for f' on the interval $[c_1, c_2]$ so that there exists $x_0 \in (c_1, c_2)$ such that

$$\frac{f'(c_2) - f'(c_1)}{c_2 - c_1} = f''(x_0)$$

$$\Rightarrow \quad f''(x_0) = \frac{f'(c_2)-f'(c_1)}{c_2-c_1}$$

$$= \frac{1}{c_2-c_1}\left[-\frac{f(c)}{b-c} - \frac{f(c)}{c-a}\right] \qquad [\because f(a) = f(b) = 0]$$

$$< 0 \qquad [\because f(c) > 0]$$

21. Let f be a twice differentiable function for all real x, $f(1)$ = 1, $f(2) = 4$ and $f(3)$ = 9. Then, which one of the following statements is definitely true?

(a) f'' (x) = f'(x) = 5 for some x ∈ (1, 3)

(b) f'' (x) = 2 for all x ∈ $(1, 3)$

(c) f'' (x) = 3 for all x ∈ $(1, 3)$

(d) f''(x) attains the value 2 for some x ∈ (1, 3)

Ans. A, Consider H(x) = f(x) – 2g(x), ∀ x ∈ $[0, 1]$.

Clearly, H (x) is differentiable in (0, 1).

Also, H(0) = f(0) – 2g(0) = 2 – 0 = 2

H(1) = f(1) – 2g(1) = 6 – 2(2) = 2

Therefore, H(0) = H(1)

Hence, by Rolle's therem, there exists c ∈ (0, 1), such that

H'(c) = 0 $\Rightarrow f'(c) - 2g'(c) = 0$

$\Rightarrow \quad f'(c) = 2g'(c)$

∴ k = 2

22. sin^{-1} (sin 5) > x^2 – 4x holds, if

(a) X = 2 - $\sqrt{9 - 2\pi}$

(b) X = 2 + $\sqrt{9 - 2\pi}$

(c) X > 2 + $\sqrt{9 - 2\pi}$

(d) X ∈ (2 - $\sqrt{9 - 2\pi}, 2 + \sqrt{9 - 2\pi}$

Ans. D, $\frac{3\pi}{2} < 5 < \frac{5\pi}{2}$

$\Rightarrow \quad sin^{-1}(sin5) = 5 - 2\pi$

Given, sin^{-1}(sin 5) > $x^2 - 4x$

$\Rightarrow \quad x^2 - 4x + 4 < 9 - 2\pi$

$\Rightarrow \quad (x-2)^2 < 9 - 2\pi$

or $\quad -\sqrt{9-2\pi} < (x-2) < \sqrt{9-2\pi}$

$\Rightarrow$ 2 - $\sqrt{9-2\pi}$ < x < 2 + $\sqrt{9-2\pi}$

23. If $(sin^{-1}x)^2 - (cos^{-1}x)^2 = a\pi^2$, then the range of a is

(a) $\left[\frac{-3}{4}, \frac{1}{4}\right]$ (b) $\left[\frac{-3}{4}, \frac{3}{4}\right]$ (c) $[-1, 1]$ (d) $\left[-1, \frac{3}{4}\right]$

Ans. A, $\because a\pi^2 = (sin^{-1}x)^2 - (cos^{-1}x)^2$

$\Rightarrow \quad a\pi^2 = (sin^{-1}x - cos^{-1}x)(sin^{-1}x + cos^{-1}x)$

$\Rightarrow \quad a\pi^2 = (sin^{-1}x - cos^{-1}x)\left(\frac{\pi}{2}\right)$

$\Rightarrow \quad 2a\pi = 2sin^{-1}x - \frac{\pi}{2}$

Now, $-\pi \le 2sin^{-1}$ x $\le \pi$

$\Rightarrow \quad -\frac{3\pi}{2} \le sin^{-1}x \le \pi$

$\therefore \quad \frac{-3\pi}{2} \le 2a\pi \le \frac{\pi}{2}$

$\Rightarrow \quad \frac{-3}{4} \le a \le \frac{1}{4}$

24. $\int \frac{x^6+1}{x^2+1}$ dx is equal to

(a) $\frac{x^5}{5} - \frac{x^3}{3} + x - 2\tan^{-1} x + c$

(b) $\frac{x^5}{5} + \frac{x^3}{3} - x - 2\tan^{-1} x + c$

(c) $-\frac{x^5}{5} + \frac{x^3}{3} - x - 2\tan^{-1} x + c$

(d) $\frac{x^7}{7} + \frac{x^5}{5} - \frac{x^3}{3} + 2\tan^{-1} x + c$

Ans. A, Let / = $\int \frac{x^6-1}{x^2+1}$dx

$= \int \left[(x^4 - x^2 + 1) - \frac{2}{x^2+1}\right] dx$ [by long division]

$= \frac{x^5}{5} - \frac{x^3}{3} + x - 2\tan^{-1} x + C$

25. $\int \frac{\sec 2x - 1}{\sec 2x + 1}$ dx is equal to

(a) ($\sec^2$ x - x) + c (b) (tan x - x) + c (c) ($\sec^2$ x + x) + c (d) (tan x + x) + c

Ans. B, Let / = $\int \frac{sec2x-1}{sec2x+1} dx$

$= \int \frac{1-cos2x}{1+\cos 2x} dx = \int \frac{1-1+2sin^2 x}{1+2\,cos^2 x-1}$ dx

$= \int \frac{sin^2 x}{cos^2 x} dx = \int tan^2\ x dx$

$= \int (sec^2 x - 1) dx = \tan x - x + C$

26. $\int \left[sin^2\left(\frac{9\pi}{8} + \frac{x}{4}\right) - sin^2\left(\frac{7\pi}{8} + \frac{x}{4}\right)\right]$ dx is equal to

(a) 2 cos $\frac{x}{2}$ + c (b) $\sqrt{2}\, cos\frac{x}{2}$ + c (c) $-\sqrt{2}\, cos\frac{x}{2}$ + c (d) None of these

Ans. C, Let / = $\int \left[sin^2\left(\frac{9\pi}{8} + \frac{x}{4}\right) - sin^2\left(\frac{7\pi}{8} + \frac{x}{4}\right)\right] dx$

$= \int sin\left(\frac{9\pi}{8} + \frac{x}{4} + \frac{7\pi}{8} + \frac{x}{4}\right). sin\left(\frac{8\pi}{8} + \frac{x}{4} + \frac{7\pi}{8} + \frac{x}{4}\right) dx$

$[\because (sin^2 A - sin^2 B) = \sin(A+B)\sin(A-B)]$

$= \int sin\left(2\pi + \frac{x}{2}\right). sin\frac{\pi}{4} dx$

$= \frac{1}{\sqrt{2}} \int sin\frac{x}{2} dx$ $[\because sin(2\pi + \theta) = sin\theta]$

$=\frac{1}{\sqrt{2}} \times 2\left(-cos\frac{x}{2}\right) + C$

= - $\sqrt{2}$ cos$\frac{x}{2}$ + C

27. $\int \frac{\cos 4x + 1}{\cot x - \tan x}$ dx is equal to

(a) $\frac{\cos 4x}{8}$ + c (b) $\frac{-\cos 4x}{4}$ + c (c) $\frac{\cos 8x}{8}$ + c (d) $\frac{-\cos 4x}{8}$ + c

Ans. D, let I = $\int \frac{\cos 4x+1}{\cot x-\tan x} dx$

= $\int \frac{2cos^2 2x-1+1}{\frac{\cos x}{\sin x}-\frac{\sin x}{\cos x}}$ [$\because$ cos 2A = 2 $cos^2 A - 1$]

= $\int \frac{2cos^2 2x(\sin x.\cos x)}{cos^2 x - sin^2 x} dx = \int \frac{cos^2 2x\ .\ sin2\,x}{\cos 2\,x}\ dx$

= $\int cos2\,x\ sin2x\ dx = \frac{1}{2}\int \sin 4x dx$

$=\frac{1}{2}\left(-\frac{\cos 4x}{4}\right) = -\frac{1}{8}\cos 4x + C$

28. $\int \frac{\sin 2x + \sin 5x - \sin 3x}{\cos x + 1 - 2\ sin^2\ 2x}$ dx is equal to

(a) -2 cos x + c (b) 2 sin x + c (c) - 2 sin x + c (d) cos x + c

Ans. A, Let I = $\int \frac{\sin 2x+\sin 5x-\sin 3x}{\cos x+1-2\ sin^2 2x} dx$

= $\int \frac{sin2x+2\sin 4x.sin}{\cos x+\cos 4x}\ dx$

$\left[\begin{array}{c}\because (sinC - \sin D) = 2\cos\frac{C+D}{2}\ sin\frac{C-D}{2} \\ and\ 1 - 2sin^2 A = cos\ 2\ A\end{array}\right]$

$=\int \frac{2\sin x\ coa\ x+2\cos 4\ x\ .\sin x}{\cos x+\cos 4x}$

= $\int \frac{2\sin x\,[\cos x+\cos 4x]}{(\cos x+\cos 4x)} dx$

= 2$\int sinx\ dx = -2\cos x + c$

29. $\int \left(\frac{cot^2 2x-1}{2\cot 2x} - \cos 8x.\cot 4x\right)$ dx is equal to

(a) $\frac{\cos 8x}{8}$ +c (b) $\frac{\sin 8x}{8}$ +c (c) -$\frac{\cos 8x}{8}$ +c (d)$-\frac{\sin 8x}{8}$ +c

Ans. C, Let I = $\int \left(\frac{cot^2\ x-1}{2\cot 2x} - \cos 8x.\cot 4x\right)\ dx$

$=\int (\cot 4x - \cos 8x.\cot 4x)\ dx$ $\left[\because cot2\ A = \frac{2\ cot^2\ A-1}{2\cot A}\right]$

= $\int \cot 4x(1-\cos 8x)dx$

= $\int \cot 4x\ 2sin^2\ 4x\ dx$ [$\because$ $(cos2\ A) = 2\ sin^2\ A$]

= $\int 2\sin 4x\ cos4xdx$

$=\int \sin 8x\ dx$ [$\because$ $2\sin A\cos A = sin2\ A$]

$=-\frac{1}{8}\cos 8x + C$

30. $\int \frac{cos^4\ x - sin^4\ x}{\sqrt{1+\cos 4x}}$ dx, (cos 2x > 0) is equal to

(a) -$\frac{x}{\sqrt{12}}$ + c (b) 2x + c (c) $\sqrt{2}$x + c (d) $\frac{x}{\sqrt{2}}$ + c

Ans. D, let I = $\int \frac{cos^4\ x-\sin 4\ x}{\sqrt{1+\cos 4x}} dx$ [$\cos 2x > 0$]

$=\int \frac{(cos^2 x)^2-(sin^2 x)^2}{\sqrt{1+\cos 4x}} dx$

$=\int \frac{(cos^2 x)^2-(sin^2 x)^2}{\sqrt{2\, cos^2 2x}} dx$ $[\because 1+\cos 2A = 2cos^2 A]$

$=\int \frac{cos^2 x - sin^2 x}{\sqrt{2}\cos 2x}$ dx $= \frac{1}{\sqrt{2}} \int \frac{\cos 2x}{cos 2x} dx$

$= \frac{1}{\sqrt{2}} \int 1 dx = \frac{1}{\sqrt{2}}\, x + c$