



Times Allowed – 3 Hours

Maximum Marks – 70

SAMPLE PAPER

MATHEMATICS

General Instruction:-

- i. All questions are **compulsory**.
- ii. Q. no. 1 to 4 are very short answer questions and carry 1 mark each.
- iii. Q. no. 5 to 12 are short answer questions and carry 2 marks each.
- iv. Q. no. 13 to 23 are also short answer questions and carry 4 marks each.
- v. Q. no. 24 to 29 are long answer questions and carry 5 marks each.
- vi. Use log tables if necessary, use of calculators is **not** allowed.

SECTION A

1. If matrices $A = \text{diag}(2, -3, 4)$ and $B = \text{diag}(4, 5, -1)$, find $A + B$.
2. If A is a 3×3 matrix and $|3A| = k|A|$, then write the value of k .
3. Find λ , when the projection of $\vec{a} = \lambda\hat{i} + \hat{j} + 4\hat{k}$ on $\vec{b} = 2\hat{i} + 6\hat{j} + 3\hat{k}$ is 4 units.
4. If the binary operation $*$ on the set Z of integers is defined by $a * b = a + 3b^2$, then find the value of $2 * 4$.

SECTION B

5. Evaluate $\int_0^{\pi} |\cos x| dx$.
6. Evaluate $\int_1^2 [x^2] dx$, where $[x]$ is greatest integer function
7. Find the angle between the following pair of lines :
 $\frac{-x+2}{-2} = \frac{y-1}{7} = \frac{z+3}{-3}$ and $\frac{x+2}{-1} = \frac{2y-8}{4} = \frac{z-5}{4}$
And check whether the lines are parallel or perpendicular.
8. Find $\frac{dy}{dx}$, if $y = \sin^{-1} \left[\frac{6x-4\sqrt{1-4x^2}}{5} \right]$.
9. If \hat{a} and \hat{b} are unit vectors inclined at an angle θ , then prove that $\sin \frac{\theta}{2} = \frac{1}{2} |\hat{a} - \hat{b}|$.
10. If A and B are two events such that $P(A) = \frac{1}{4}$, $P(B) = \frac{1}{2}$ and $P(A \cap B) = \frac{1}{8}$. Find $P(\text{not } A \text{ and not } B)$.
11. Find $|\vec{a} - \vec{b}|$, if two vectors \vec{a} and \vec{b} are such that $|\vec{a}| = 2$, $|\vec{b}| = 3$ and $\vec{a} \cdot \vec{b} = 4$

12. Three cards are drawn successively with replacement from a well shuffled pack of 52 cards. Find the probability distribution of the number of spades. Hence find the mean of the distribution.

SECTION C

13. If $f(x) = \sqrt{x^2 + 1}$, $g(x) = \frac{x+1}{x^2+1}$ and $h(x) = 2x - 3$, then find $f'(h'(g'(x)))$.

14. Using properties of determinants, show that ΔABC is isosceles if :

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 + \cos A & 1 + \cos B & 1 + \cos C \\ \cos^2 A + \cos A & \cos^2 B + \cos B & \cos^2 C + \cos C \end{vmatrix} = 0$$

15. Prove that the curves $y^2 = 4ax$ and $xy = c^2$ cut at right angles if $c^4 = 32a^4$.

OR

Separate the interval $\left[0, \frac{\pi}{2}\right]$ into subintervals in which $f(x) = \sin^4 x + \cos^4 x$ is strictly increasing or decreasing.

16. If $x = a(\cos t + t \sin t)$ and $y = a(\sin t - t \cos t)$, then find $\frac{d^2y}{dx^2}$ at $t = \frac{\pi}{4}$.

17. Evaluate $\int \sqrt{\tan x} dx$.

18. Consider $f: R \rightarrow [-5, \infty)$ given by $f(x) = 9x^2 + 6x - 5$, where R^+ is the set of all non-negative real numbers. show that f is invertible with $f^{-1}(y) = \frac{\sqrt{y+6}-1}{3}$.

OR

Let N be the set of all natural numbers. A relation R be defined on $N \times N$ by $(a, b)R(c, d) \Leftrightarrow a + d = b + c$. Show that R is equivalence relation.

19. A variable plane is at a constant distance p from the origin and meets the coordinate axes in A, B, C . show that the locus of the centroid of the tetrahedron $OABC$ is $x^{-2} + y^{-2} + z^{-2} = 16p^{-2}$.

20. Prove that : $\sin^{-1}\left(\frac{12}{13}\right) + \cos^{-1}\left(\frac{4}{5}\right) + \tan^{-1}\left(\frac{63}{16}\right) = \pi$

21. A point on the hypotenuse of a right angle triangle is at distance a and b from the sides. Show that the minimum length of the hypotenuse is $(a^{2/3} + b^{2/3})^{3/2}$.

OR

Find the maximum area of the isosceles triangle inscribed in the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with its vertex at one end of major axis.

22. Evaluate the following : $\int_{-1}^{3/2} |x \sin \pi x| dx$.

OR

Evaluate:

$$\int_0^1 \cot^{-1}(1 - x + x^2) dx$$

23. Evaluate $\int \frac{x^2+1}{(x-1)^2(x+3)} dx$.

Section D

24. Show that the differential equation:

$(x^2 + 2xy - y^2) dx + (y^2 + 2xy - x^2) dy = 0$ is homogeneous and solve it.

25. Find the equation of the plane passing through the line of intersection of the planes

$2x + y - z = 3$, $5x - 3y + 4z + 9 = 0$ and parallel to the line $\frac{x-1}{2} = \frac{y-3}{4} = \frac{z-5}{5}$.

26. Two schools P and Q want to award their selected students on the values of discipline , politeness and punctually. The school P wants to award Rs. x each , Rs. y each and Rs. Z each for the three respective values to its 3, 2, and 1 students with a total award money of Rs. 1,000. School Q wants to spend Rs. 1500 to award its 4, 1 and 3 students on the respectively values (by giving the same award money for the three values as before). If the total amount of awards for one prize on each value is Rs. 600, using matrices find the award money for each value. A part from the above three values , suggest one more values for awards.
27. In answering a question on a multiple choice, a student either knows the answer or guesses. Let $\frac{3}{4}$ be the probability that he knows the answer and $\frac{1}{4}$ be the probability that he guesses. Assuming that a student who guesses at the answer will be incorrect with probability $\frac{1}{4}$. What is the probability that the student knows the answer, given that he answered correctly?
28. Using integral, find the area of the following region:
 $\{(x, y): |x - 1| \leq y \leq \sqrt{5 - x^2}\}$
29. A small firm manufactures gold rings and chains. The total number of rings and chains manufactures per day is at most 24. It takes 1 hour to makes a ring and 30 minutes to makes a chain. The maximum number of hours available per day is 16. If the profit on a ring is Rs.300 and that on a chain is Rs.190, find the number of rings and chains that should be manufactured per day, so as to earn the maximum profit. Makes it as L.P.P. and solve it graphically.