Mathematics Question Paper

Drop.

1. If the roots of $ax^2 + bx + c = 0$ are of the form $\frac{a}{a-1}$ and $\frac{a+1}{a}$, then the value of $(a+b+c)^2$ is

   (a) $b^2 - 4ac$ (b) $b^2 - 2ac$ (c)$2b^2 - ac$ (d)$4b^2 - 2ac$

Sol. $\frac{a}{a-1} + \frac{a+1}{a} = \frac{-b}{a}$ and $\frac{a}{a-1} . \frac{a+1}{a} = \frac{c}{a}$

$\Rightarrow \quad \frac{2a^2-1}{a^2-a} = -\frac{b}{a}$ (1)

And $\frac{a+1}{a-1} = \frac{c}{a}$

From Equation (2) a$a + a = ca - c$

$\Rightarrow$ a(c - a) = a + c $\Rightarrow$ a = $\frac{c+a}{a-a}$

Putting in Equation (i) $\frac{2\left(\frac{c+a}{c-a}\right)^2 - 1}{\left(\frac{c+a}{c-a}\right)^2 - \left(\frac{c+a}{c-a}\right)} = -\frac{b}{a}$

$\Rightarrow \frac{2(c+a)^2-(c-a)^2}{(c+a)^2-(c^2-a^2)} = -\frac{b}{a}$

$\Rightarrow \frac{(c+a)^2+4ac}{2a^2+2ac} = -\frac{b}{a}$

$\Rightarrow (c+a)^2 + 4ac = -2b(a+c)$

$\Rightarrow (c+a)^2 + 2b(a+c) + 4ac = 0$

$\Rightarrow (c+a)^2 + 2b(a+c) + b^2 = b^2 - 4ac$

$\Rightarrow (c+a+b)^2 = b^2 - 4ac$

Hence, $(a+b+c)^2 = b^2 - 4ac$

2. If $2^{x^2} : 2^{2x} = 8^k : 1$, then the equation has only one solution, if

   (a) $k = \frac{1}{3}$ (b) $k = \frac{-1}{3}$ (c) $k > \frac{1}{3}$ (d) $k < \frac{-1}{3}$

Sol. $2^{x^2} = 2^{2x} . 2^{3k} = 2^{2x+3k}$

$\Rightarrow \quad x^2 = 2x + 3k \Rightarrow x^2 - 2x - 3k = 0$

D = 4 + 12k = 0 $\Rightarrow$ k = $\frac{-1}{3}$

3. If conjugate and reciprocal of a complex number z = x + iy are equal, then

   (a) $x + y = 1$ (b)$x^2 + y^2 = 1$ (c)$x = 1$ and $y = 0$ (d)$x = 0$ and $y = 1$

Sol. Given: Complex number (z) x + iy and conjugate of z = ($\bar{z}$) = x – iy. Since reciprocal of the complex number (z) and its conjugate ($\bar{z}$) are its conjugate ($\bar{z}$)a re equal, therefore $\bar{z} = \frac{1}{z}$

or $\quad z\bar{z} = 1$ $\quad$ or $\quad (x+iy)(x-iy) = 1$

or $\quad x^2 - (iy)^2 = 1$ or $\quad x^2 + y^2 = 1.$

4. If $|z| = 1$ and $z \neq \pm 1$, then all the value of $\frac{z}{1-z^2}$ lie on

   (a) A line not passing through the origin

   (b) $|z| = \sqrt{2}$

   (c) The x-axis

   (d) The y-axis

Sol. Since $|z| = 1$, therefore z = $e^{i\theta}$ as $r = 1$

Therefore, $\frac{z}{1-z^2} = \frac{1}{z^{-1}-z} = \frac{1}{e^{-i\theta}-e^{i\theta}} = \frac{-1}{2i\sin\theta}$

= 0 + I . $\frac{1}{2\sin\theta}$ $\quad \left[\because \frac{1}{i} = -i\right]$

Thus, real part of $\frac{z}{1-z^2}$ is zero. Hence it lies on y-axis.

5. Let the positive numbers a, b, c, d, be in AP. Then abc, abd, acd, bcd are

(a) Not in AP/GP / HP (b) in. AP (c) in GP (d) in HP

Sol. a, b, c, d are in AP

Therefore, d, c, b, a are also in AP

$\Rightarrow$ $\frac{d}{abcd}, \frac{c}{abc}, \frac{1}{acd}, \frac{1}{bcd}$ are in AP

$\Rightarrow$ $\frac{1}{abc}, \frac{1}{abd}, \frac{1}{acd}, \frac{1}{bcd}$ $are\ in\ AP$

$\Rightarrow$ abc, abd, acd, bcd are in HP

6. The sum of r terms of an AP is denoted by s, and $\frac{s_m}{s_n} = \frac{m^2}{n^2}$. Find the ratio of the m[th] terms and the n[th] term of the AP

(a) $\frac{2m-1}{2n-1}$ (b) $\frac{2m+1}{2n-1}$ (c) $\frac{2m-1}{2n+1}$ (d) $\frac{2m+1}{2n+1}$

Sol. let the first term = a and the common difference = d

Therefore, $s_m = \frac{m}{2}[2a + (m-1)d]$ and $s_n = \frac{n}{2}[2a + (n-1)d]$

Therefore, $\frac{s_m}{s_n} = \frac{\frac{m}{2}[2a+(m-1)d]}{\frac{n}{2}[2a+(n-1)d]}$ or

$\frac{m^2}{n^2} = \frac{m}{n} \cdot \frac{2a+(m-1)d}{2a+(n-1)d}$

Or $\frac{2a+(m-1)d}{2a+(n-1)d} = \frac{m}{n}$.

This is an identity in m and n.

Thus, putting 2m – 1 for m and 2n – 1 for n we get

$\frac{2a+(2m-1-1)d}{2a+(2n-1-1)d} = \frac{2m-1}{2n-1}$ or $\frac{a+(m-1)d}{a+(n-1)d} = \frac{2m-1}{2n-1}$

Or $\frac{m^{th}\ term}{n^{th}\ term} = \frac{2m-1}{2n-1}$

7. The number of words that can be formed with the letters of the word 'PARALLEL' so that all L's do not come together but both A come together is

(a) 420 (b) 3000 (c) 720 (d) none of these

Sol. When both A are together, taking them as single identity,

No. of ways = $\frac{7!}{3!}$ $(\because L's\ are\ thrice)$

When both A and all three L's are together, taking term as two separate identity. No. of ways = 5!

Required ways $\frac{7!}{3!} - 5! = 840 - 120 = 720$

8. The number of arrangements of the letters of the word 'BANANA' in which the two N' s do not appear adjacently is

(a) 40 (b) 60 (c) 80 (d) 100

Sol. Total – together (sting method)

$3A^s, 2N^s, 1B$

= $\frac{6!}{3!2!} - \frac{5!2!}{3!2!} = 60 - 20 = 40$

9. The total number of arrangements of the letters in the expansion $a^3b^2c^4$ when written at full length is

(a) 1260 (b) 2520 (c) 610 (d) none of these

Sol. There are 9 letters viz., 3a's, 2b's and 4c's.

These can be arranged in $\frac{9!}{3!2!4!}$ $i.e.$, 1260 ways

10. The total number of four digit odd number that can be formed using 0, 1, 2, 3, 5, 7 are

(a) 400 (b)720 (c) 375 (d)216

Sol. Digits are 0, 1, 2, 3, 5, 7

The last place can be filled by $(1, 3, 5, 7)$ 4 ways as the number is to be odd and first place can be filled by 5 ways (excluding 0), so total number will be

5 × 6 × 6 × 4 = 720

(second and third places can be filled by 6 ways)

11. The sum of the digits in the units place of all numbers formed with the help of 3, 4, 5, 6 taken all at a time is

(a) 246 (b)252 (c) 6 (d) none of these

Sol. When number at unit place is 3, then other three numbers can be arranged in 3! ways.

Therefore, the sum of the digits in units place when 3 is their at unit place = 3! × 3

Similarly, the sum of the digits in

4 at unit place = 3! × 4

5 at unit place = 3! × 5

6 at unit place = 3! × 6

Thus, the sum of the digits in the unit place of all numbers formed with the help of 3, 4, 5, 6 taken all at a time is

$$(3! \times 3) + (3! \times 4) + (3! \times 5) + (3! \times 6)$$

$$=3!\,(3 + 4 + 5 + 6) = 6 \times 18 = 108$$

12. For the curve $xy = c^2$ $the\ subnormal\ at\ any\ point\ varies\ as$:

(a) x^3 (b) x^2 (c) y^3 (d) ∞

Sol. Given, $y = \frac{c^2}{x}$

$\Rightarrow \qquad \frac{dy}{dx} = c^2\left(-\frac{1}{x^2}\right)$

∴ Subnormal at any point $= y\frac{dy}{dx}$

$$= y \times \left(\frac{c^2}{x^2}\right) = \frac{-y^3}{c^2}$$

∴ Subnormal ∝ y^3.

13. Tangent is drawn to ellipse $\frac{x^2}{27} + y^2 = 1$ at $(3\sqrt{3}\cos\theta, \sin\theta)$ $(where\ \theta \epsilon\ (0, \pi/2))$. Then the value of θ such that sum of intercepts on axes made by this tangent is minimum, is:

(a) $\pi/3$ (b) $\pi/6$ (c) $\pi/8$ (d) $\pi/4$

Solution:

Equation of tangent at $(3\sqrt{3}\cos\theta, \sin\theta)$ is:

$$\frac{x\cos\theta}{3\sqrt{3}} + \frac{y\sin\theta}{1} = 1$$

Thus, sum of intercepts $= (3\sqrt{3}\cos\theta + cosec\ \theta) = f\ (\theta)\ [say]$

$\Rightarrow \quad f'(\theta) = \frac{3\sqrt{3}\ sin^3\ \theta - cos^3\theta}{sin^2\ \theta\ cos^2\ \theta}$.

Put $f'(\theta) = 0$

$\therefore \qquad sin^3\theta = \frac{1}{3^{3/2}}\ cos^3\theta$

$\Rightarrow \qquad \tan\theta = \frac{1}{\sqrt{3}} \quad \Rightarrow \quad \theta = \frac{\pi}{6}$

Also, for $0 < \theta < \frac{\pi}{6}, \frac{dz}{d\theta} < 0\ \ and\ \ for\ \frac{\pi}{6} < \theta < \frac{\pi}{2}, \frac{dz}{d\theta} > 0$

∴ Minimum at $\theta = \frac{\pi}{6}$

14. For a particles moving in a straight line, if time t be regarded as a function of velocity v, then the rate of change of the acceleration a is given by:

(a) $a^2\frac{d^2t}{dv^2}$ (b) $a^3\frac{d^2t}{dv^2}$ (c) $-a^3\frac{d^2t}{dv^2}$ (d) None of these

Solution:

Let $t = f(v), then$

$$\frac{dt}{dv} = f'(v) \quad \Rightarrow \quad \frac{dv}{dt} = \frac{1}{f'(v)}$$

$$\Rightarrow \qquad a = \frac{1}{f'(v)} \quad \Rightarrow \quad af'(v) = 1$$

$$\Rightarrow \qquad af''(v)\frac{dv}{dt} + \frac{da}{dt}\ f'(v) = 0$$

$$\Rightarrow \qquad a^2 f''(v) + \frac{da}{dt} \times \frac{1}{a} = 0 \ \Rightarrow \frac{da}{dt} = \ -a^3\ \frac{d^2t}{dv^2}$$

15. A point is moving on $y = 4 - 2x^2$. The $x - coordinate\ of$ the point is decreasing at the rate of 5 unit per second. Then, the rate at which $y - coordinate$ of the point is changing when the point is at (1, 2) is:

(a) 5 units (b) 10 units (c) 15 units (d) 20 units

Solution:

Given equation of curve is

$$y = 4 - 2x^2$$

$$\Rightarrow \qquad \frac{dy}{dt} = -4x\frac{dx}{dt}$$

Given, $\frac{dx}{dt} = -5, at\ point\ (1,\ 2)$

$\therefore \qquad \frac{dy}{dt} = \ -4\ (1)\ (-5) = 20$ unit/s

16. If $f(x)\ is\ a\ twice\ differentiable\ function, then\ between\ two\ consicutive\ roots\ of\ the\ equation\ f'(x) = 0$, then exists

(a) At least one roots of $f\ (x) = 0$

(b) At most one roots of $f(x) = 0$

(c) Exactly one root of $f(x) = 0$

(d) At most one root of $f''(x) = 0$

Solution: (b)

If $b - 1\ \leq x \leq b, from\ Rolle's\ theorem$

$$f(b-1) = f(b) = 0$$

and $\qquad f'(x) = 0$

Suppose $\qquad x = 0$

such that $\qquad b - 1 \leq c \leq b$

Hence, $f\ (x) = 0\ has\ ata\ most\ one\ root.$

17. A function $f\colon\ R\ \rightarrow R$ satisfies the equation $f(x)\ f(y) - f(xy) = x + y, \forall\ x, y\ \epsilon\ R\ and\ f(1) > 0, also\ h(x) = f(x)\ f^{-1}(x)$, then length of longest interval in which $h\ (\sin x + \cos x)$ is strictly increasing is:

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{2}$ (d) π

Solution: (c)

$f(x)\ \ f(y) -\ f(xy) = x + y, \forall\ x, y\ \epsilon\ R$ ....(i)

Substituting $\ x = 1,\ \ y = 1, we\ get$

$$\left(f(1)\right)^2 -\ f(1) - 2 = 0$$

$$(f\ (1) - 2)\ (f(1) + 1) = 0$$

As, $\qquad f(1) > 0 \Rightarrow f(1) = 2$

Now, substituting $y = 1\ in\ Eq. (i), we\ get$

$$f(x)\ \times 2 - f(x) = x + 1$$

$\therefore \qquad f(x) = x + 1$

$$f^{-1}\ (x) = x - 1$$

$$h(x) = f(x)\ \ f^{-1}(x) = x^2 - 1$$

$\therefore h(\sin x + \cos x) = (\sin x + \cos x) - 1 = \sin 2x$

As longest interval in which $\sin x$ *is strictly increasing is* π.

$\Longrightarrow$ *For* $\sin 2x$ *it will be* $\frac{\pi}{2}$.

18. The period of the function $f(x) = 7\cos(3x+5)$ is

[a] 2π [b] $\frac{2\pi}{3}$ [c] $\frac{\pi}{3}$ [d] None of these

Solution: - [b]

$\cos x$ is a periodic function with period 2π, therefore $\cos(3x+5)$ with be a periodic function with period $\frac{2\pi}{3}$.

Hence, (b) is the correct answer.

19. The curve $x^3 - 3xy^2 = a$ and $3x^2y - y^3 = b$, where a and b are constants, cut each other

[a] at an angle $\frac{\pi}{3}$ [b] at an angle $\frac{\pi}{4}$ [c] orthogonally [d] none of these

Solution: - [c]

The two curves are

$$x^3 - 3xy^2 = a \quad \text{...(i)}$$

$$and \quad 3x^2y - y^3 = b \quad \text{...(ii)}$$

On differentiating Eq.(i) w.r.t. x, we get

$$3x^2 - 3y^2 - 6xy\frac{dy}{dx} = 0$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{x^2-y^2}{2xy}$$

On differentiating Eq. (ii) w.r.t. x, we get

$$6xy + 3x^2\frac{dy}{dx} - 3y^2\frac{dy}{dx} = 0$$

$$\Rightarrow \quad \frac{dy}{dx} = -\frac{2xy}{x^2-y^2}$$

The product of $\frac{dy}{dx}$ for the two curves

$$= \left(\frac{x^2-y^2}{2xy}\right) \times \left(\frac{-2xy}{x^2-y^2}\right) = -1.$$

The curves cut each other orthogonally.

Hence (c) is the correct answer.

20. The incentre of the triangle with verticles $(1,\sqrt{3})$, (0,0) and (2,0) is :

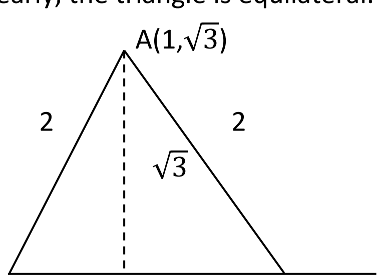
a. $\left(1,\frac{\sqrt{3}}{2}\right)$

b. $\left(\frac{2}{3},\frac{1}{\sqrt{3}}\right)$

c. $\left(\frac{2}{3},\frac{\sqrt{3}}{2}\right)$

d. $\left(1,\frac{1}{\sqrt{3}}\right)$

Sol. (d) clearly, the triangle is equilateral.



60^0 (1,0)

B(0,0) 2 C(2,0) X

So, the incentere is the same as the centroid.

$\therefore$ incentre = $\left(\frac{1+0+2}{3}, \frac{\sqrt{3}+0+0}{3}\right) = \left(1, \frac{1}{\sqrt{3}}\right)$

21. The number of terms common to two A.P.'s 3, 7, 11, …, 407 and 2, 9, 16, …, 709 is :

(a) 14 (b) 21 (c) 28 (d) none of these.

Solution : (a) Let m and n be the number of terms of terms of the A. P.'s 3, 7, 11, …, 407 and 2, 9, 16, ….709 respectively.

$\therefore \quad 407 = 3 + (m-1)(4)$ and $709 = 2 + (n-1)(7)$

$\Rightarrow 4m - 1 = 407$ and $7n - 5 = 709$

$\Rightarrow \quad 4m = 408$ and $7n = 714$

$\Rightarrow \quad m = 102$ and $n = 102.$

Thus each A.P. has 102 terms.

Let pth term of first A.P. $=$ qth terms of second A.P.

$\Rightarrow 3 + (p-1)(4) = 2 + (q-1)7$

$\Rightarrow \quad 4p - 1 = 7q - 5 \Rightarrow 4p + 4 = 7q$

$\Rightarrow \quad q = \frac{4(p+1)}{7} \Rightarrow \quad \frac{p+1}{7} = \frac{q}{4} = k(say)$

$\therefore p + 1 = 7k$ and $q = 4k$

$\Rightarrow \quad p = 7k - 1$ and $q = 4k.$

Clearly $p \leq 102$ and $q \leq 102$

$\Rightarrow 7k - 1 \leq 102$ and $4k \leq 102$

$\Rightarrow \quad 7k \leq 103$ and $k \leq 25\frac{1}{2}$

$\Rightarrow \quad k \leq 14\frac{5}{7}$ and $k \leq 25\frac{1}{2}$

$\Rightarrow \quad k = 14.$

Hence there are 14 common terms.

22. If H is the harmonic mean between P and Q, then $\frac{H}{P} + \frac{H}{Q}$ is**:**

(a) 2 (b) $\frac{P+Q}{PQ}$ (c) $\frac{PQ}{P+Q}$ (d) none of these

Solution: (a) Here $\text{H} = \frac{2PQ}{P+Q}$

$\therefore \frac{H}{P} + \frac{H}{Q} = H = \left(\frac{1}{P} + \frac{1}{Q}\right)$

$= \frac{2PQ}{P+Q}\left(\frac{Q+P}{PQ}\right) = 2.$

23. If $|z_1 - 1| < 1, |z_2 - 2| < 2, |z_3 - 3| < 3$, then $|z_1 + z_2 + z_3|$:

(a) Is less than 6 (b) is more than 3

(c) is less than 12 (d) lies between 6 and 12.

Solution : (c) $|z_1 + z_2 + z_3| = |(z_1 - 1) + (z_2 - 2) + (z_2 - 2) + 6|$

$\leq |z_1 - 1| + |z_2 - 2||z_2 - 2| + 6$

$< 1 + 2 + 3 + 6 = 12.$

24. If $log_{0.3}(x-1) < log_{09}(x-1)$ then x lies in the interval

(a) $(2,\infty)$ (b) $(1,2)$ (c) $(2,-1)$ (d) None of these

Solution : (a).

First note that $(x-1)$ must be greater than 0, that is $x > 1$.

Now $\quad log_{\,0.3}(x-1) < log_{.09}(x-1)$

$\Rightarrow \quad log_{0.3}(x-1) < log_{(0.3)^2}(x-1)$

$\Rightarrow \quad log_{0.3}(x-1) < \frac{1}{2}log_{0.3}(x-1)$

$\Rightarrow \quad 2log_{0.3}(x-1) < log_{0.3}(x-1)$

$\Rightarrow \quad log_{0.3}(x-1)^2 < log_{0.3}(x-1)$

$\Rightarrow \quad (x-1)^2 > x-1$

[Note that the inequality is reversed since the base lies between 0 and 1]

$\Rightarrow \quad (x-1)^2 - (x-1) > 0$

$\Rightarrow \quad (x-1)(x-2) > 0$

Since $x > 1$, the inequality (1) will hold if $x > 2$, that is if x lies in the interval $(2,\infty)$.

25. The value of $\frac{C_0}{1} + \frac{C_2}{3} + \frac{C_4}{5} + \ \dots\dots is$

(a) $\frac{2^n}{n+1}$ (b) $\frac{2^n - 1}{n+1}$ (c) $\frac{2^{n+1}}{n+1}$ (d) none of these

Solution :-

We have

$(1+x)^n = \sum_{r=0}^{n} C_r X^r$ ....(i)

$(1-x)^n = \sum_{r=0}^{n} (-1)^r C_r X^r$ .....(ii)

Adding equation (i) and (ii) we have

$(1+x)^n + (1-x)^n = C_0 + C_2x^2 + C_4x^4 + \dots\dots$

Integrating both sides with respect to x from 0to1 , we get

$\frac{2^{n+1}}{n+1} = \frac{C_0}{1} + \frac{C_2}{3} + \frac{C_4}{5} + \cdots$

26. There are p + q + r books in which there are p copies of the same title, q copies of another title and one copy each of r different titles. The number of ways in which one or more books can be selected is –

(a) $2^{p+q+r} - 1$ (b) $(p+1)(q+1)2^r - 1$ (c) $2^{p+q-1}(2^r - 1)$ (d) $2^{p+q}(2^r + 1) - 1$

Solution :- (b)

Out of the p books that are alike. m$(0 \le m \le p)$ books can be selected in only one way, so that the total number of selection of no books out of p copies of the same title is p + 1.

Similarly the total number of selection of no books or more books out of q copies of the same title is q + 1. The remaining r books are of r different titles and the total number of selection of no book or one or more books is 2^r as each book can be dealt with in two ways: rejections or acceptance.

The total number of selections is $(p + 1)(q + 1)\,2^r - 1$ as total rejection is excluded.

27. The number of 5 digit numbers of the form xyzyz in which x < y is

(a) 350 (b) 360 (c) 380 (d) 390

Solution :- (b)

The first digit x can be any one of 1 to 8 whereas z can be any one of 0 to 9 .

When x is 1, y can assume the values 2 to 9;

When x is 2, y can assume the values 3 to 9 and so on.

Thus the total number = (8 + 7 +.........+1) $\times = \frac{8.9}{2}.10 = 360.$

28. The straight lines $4x - 3y - 5 = 0, x - 2y - 0 = 0, 7x + y - 40 = 0\ nd\ x + 3y + 0 = 0\ from$ the sides of a:

(a) Quadrilateral (b) cyclic quadrilateral

(c) Rectangle (d) parallelogram

Solution:- (b)

Slopes of the lines re 4/3, ½, -7 and -1/3 respectively. if $\alpha\ is\ the$ angle between second and fourth.

$\tan\alpha = -1, \tan\beta = 1 \implies \alpha = 135^0, \beta = 45^0 \implies \alpha + \beta = 180^0$

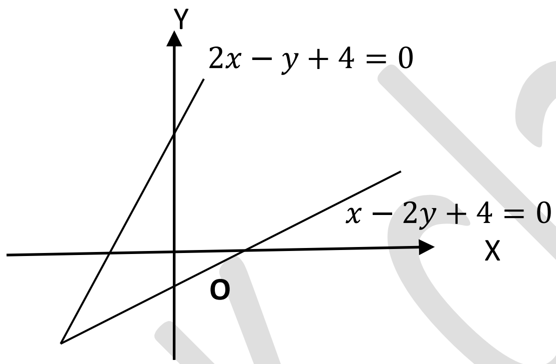
So the quadrilateral in cyclic.

Since no two sides are parallel, it cannot be a parallelogram or a rectangle.

29. The equation of the bisector of the acute angle between the lines $2x - y + 4 = 0$ and $x - 2y = 1$ is

(a) $x + y + 5 = 0$ (b) $x - y + 1 = 0$ (c) $x - y = 5$ (d) none of these

Solution :- (b)



Clearly from the figure, the origin is contained in the acute angle. Writing the equations of the lines as $2x - y + 4 = 0\ and - x + 2y = 0$, the required bisector is

$$\frac{2x-y+4}{\sqrt{5}} = \frac{-x+2y+1}{\sqrt{5}}$$

30. If the equation of the pair of straight lines passing through the point (1,1), one making an angle θ with the positive direction of x –axis and the other making the same angle with the positive direction of y –axis is $x^2 - (a+2)xy + y^2 + a(x + y - 1) = 0, a \neq -2$, then the value of sin 2 θ is:

(a) $a - 2$ (b) $a + 2$ (c) $2/(a+2)$ (d) $2/a$

Solution:- (c)

Equation of the given lines are

$y - 1 = \tan\theta\ (x - 1)\ and\ y - 1 = \cot\theta\ (x - 1)$ so their joint equation is:

$[(y-1) - \tan\theta\ (x-1)]\ [(y-1) - \cot\theta\ (x-1)] = 0$

$\implies (y-1)^2 - (\tan\theta + \cot\theta)\ (x-1)(y-1) + (x+1)^2 = 0$

$\implies x^2 - (\tan\theta + \cot\theta)\ xy + y^2 + (\tan\theta + \cot\theta - 2)\ (x + y - 1) = 0$

Comparing with the given equation we get

$\tan\theta + \cot\theta = a + 2$

$\Rightarrow \frac{1}{\sin\theta\ \cos\theta} = a + 2 \implies \sin 2\theta = \frac{2}{a+2}$